INTERREGIONAL TOTAL FLOW: A CONCEPT AND APPLICATION TO A U.S. INPUT-OUTPUT MODEL

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Introduction

The Leontief inverse is the cornerstone of traditional Input-Output Analysis. This matrix, whose properties have been exhaustively cataloged, is a linear transformation which maps a final demand vector into an output vector. As such, Input-Output Analysis is well suited to the Keynesian spirit of much macroeconomic theory and policy. It is worthwhile to understand how demand affects sectoral outputs; with such knowledge, demand management can be used in the practice of rational economic planning. Furthermore, in a market economy, where consumers signal to producers the desirability of particular commodities, the final demand orientation—implicit in many applications of Input-Output Analysis—rests on secure, theoretical ground.

Input-output relations are deeper, however, than the Leontief inverse seems to suggest. Consider, for example, some arbitrary i - j element from an arbitrary inverse. It is well known that this element represents the direct and indirect transfer from sector i to sector j necessary for the satisfaction of one unit of final demand for the goods of sector j . This transfer, when multiplied by the appropriate quantity of final demand, j , represents a sum of direct and indirect i - j input flows. Therefore, it is often and incorrectly interpreted as the total flow from i to j . But the definition itself should alert one to the fact that this flow is a total flow only with respect to the final demand j for goods from sector i . Since not all goods from sector j are directly delivered to final demand, it stands to reason that more than the Leontief "total" flow from sector i is necessary in order to maintain the integrity of sector j 's other interindustrial linkages. Hence, the Leontief total is not a true

total, but a partial one, representing only that part necessary for the fulfillment of final demand.

Since the real total flow is greater than the Leontief "total flow," it becomes imperative to ascertain and state relevant distinctions. Without a comprehensive description of total flow, it is impossible to know the actual relationships among sectors. All that can be gleaned is a limited perception, one which is entirely demand-dependent, with information that does not reflect the full complexity of input-output connections. A new total flow description is needed, then, to reveal the complete input-output mechanism and thereby show the true pattern of intersectoral relationships and dependencies. Only in this way can the importance of any one sector to the whole system be known.

Impact analysis, as currently practiced, relies almost exclusively on a Leontief format, which implies that the impacts assessed derive from changes in final demand. An economy, however, is subject to exogenous forces which are not just consumption-related. An example that will be fully described in the sequel is the phenomenon whereby sectors, or regions, externalize their inputs to sources outside the system, as cost considerations begin to favor exogenous markets. Analysis developed from a total flow concept can be used to assess the impact of such alternative input scenarios, as well as impacts issuing from other exogenous forces.

Previous attempts to characterize a total flow concept have been largely limited to single sector impacts. In a seminal input-output application, Isard and Kuenne (1953) developed a measure of the total regional impact generated by the appearance of a new steel plant. Although such an approach and others like it (Little and Doeksen, 1968) yield valuable information, they do not generalize the principle of

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total flow into a concept of global applicability. In this paper, an initial step is taken to provide a foundation for developing such a concept. In addition, a variant of Input-Output Analysis based on total flow considerations is introduced; for the sake of distinction it is called Flow Analysis.

In what follows, the basic concept together with a methodology of Flow Analysis are first presented. It will be seen that such a framework can be used to analyze an input-output system in new ways, which provide interesting and useful insights. Once the underlying principles are made clear, an application is undertaken in order to further clarify total flow, and to indicate the potency of Flow Analysis.

The application involves an analysis of interregional data for the United States collected and published by Polenske (1980). In what follows, a region becomes, in function, a "sector," and trade between regions is analogous to transfers between sectors. Final demand is domestic demand in each region, as well as exports to the global economy. In selecting the national scale with an interregional partition as an analytical framework, we hope to answer several questions germane to the study of regional interactions in a national economy. These questions are:

- What are the true total flows between regions?
- How much of the total flow is measured by Leontief Input-Output Analysis?
- What is the nature of interregional linkages in the U.S.?

Flow Analysis

An input-output structure can be perceived as a network of flows which enter the system as "value added," circulate as "intermediate transactions," and exit as "final demand." This suggests two ways of viewing the system. One is from the perspective of some flow entering the system as value added or entering a sector as intermediate input. The other is from the point of view of intermediate output or of final demand. This paper develops the flow concept from the demand, or output, side of the general model. It should be noted that a similar development may be pursued from the supply side as well.¹ We have chosen to work with the demand side in order to maintain parallels with conventional Input-Output Analysis. We presume, as well, the usual Leontief assumptions of linear-

ity between inputs and outputs and fixed-coefficient technology.

In economics, flow approaches to Input-Output Analysis are not widespread. Work has been done using flow concepts on the propagation of errors in technical coefficient matrices (Evans, 1954; Jensen and West, 1980) and on measures of interconnectedness (Yan & Ames, 1965; Robinson & Markandya, 1973; Blin & Murphy, 1974; Hamilton & Jensen, 1983). Generally, a "black box" attitude pervades this research, in that the input-output economy is considered a closed system on which final demand impinges from outside.

A flow approach adapted to the interior of an input-output system has been developed in ecology, a field in which no force comparable to final demand affects the system of interest. In ecology, Input-Output Analysis is used to describe the relationships between components, or "sectors," of ecosystems. An internal, flow oriented point of view is assumed and elaborated, a priori, by theorists in this field. Notions of "diversity," "complexity" and "stability"—described by suitable flow relations—are important to ecologists as well as to policy makers concerned with environmental impact. The approach developed in this paper has been influenced by the ecological adaptation of Input-Output Analysis (Hannon, 1973; Patten et al., 1976; Finn, 1976; Ulanowicz & Kemp, 1979; Ulanowicz, 1980; Bosserman, 1981).

Flow Matrices

In proceeding with the conceptual development, we must first define 3 matrices. These are:

(i) *The direct-flow matrix:*

$Z = [z_{ij}]$ - the standard $n \times n$ transactions matrix representing intermediate transfers within a system divided into n sectors;

(ii) *The total-output-destination matrix:*

$$Z^{\text{dest}} = [z^{\text{dest}}_{ij}] = (I - A)^{-1}\hat{Y},$$

where I - the identity matrix,

A - the technical coefficients matrix,

$\hat{Y}$ - the diagonal matrix created from the final demand vector, y ; and,

(iii) *The exit flow matrix:*

$$Z^{\text{exit}} = Z^{\text{dest}} - \hat{Y}.$$

The Z^{dest} matrix expresses total (direct + indirect) inputs from sector i , embodied in final demand commodities j , y_j . Each entry of this

matrix represents the portion of total output from sector i which leaves the system through sector j .²

The Z^{exit} matrix represents the final destinations of intermediate flows, which link particular system sectors to one another. This matrix is obtained by eliminating "external flows," or deliveries to final demand outside the input-output system.

Observe the following equalities:

$$x = Zi + y = Z^{\text{exit}}i + y = Z^{\text{dest}}i,$$

where i is the unit vector and x is the vector of total sectoral outputs. Each of the above three matrices is founded on the same set of flows, but in a different way. Hence, each depicts a different aspect of system function. The Z matrix indicates the volume of flow moving directly from i to j . The Z^{exit} matrix measures how much intermediate outflow from i will exit the system through j . Finally, the elements of Z^{dest} represent the total outflow from i exiting through j .

Total Flow

Given an open, static input-output system for a national economy with domestic flows:

$$x = (I - A)^{-1}y,$$

consider what transpires should an arbitrary sector, k , begin receiving inputs from sources outside the system, say, through international trade. It is important to realize that such an eventuality need not lead to the sector k complete withdrawal from economic life of the system, as it might still produce its previous outputs. In fact, this is an assumption we make, namely, that all outputs from the sector are delivered as before despite new input sources. The numerical implication of such an "externalization"³ is that the k -th column of the technical coefficients matrix, A , is replaced by a vector of zeroes. Call this new matrix A_k . If a Leontief inverse is calculated from this column-reduced table, and if the original final demand vector, y , is applied to the altered system, an output vector is derived, one with interesting properties. This vector, x_k , is defined simply as:

$$x_k = (I - A_k)^{-1}y.$$

Note that this is the output which would be realized if sector k no longer received inputs from the economy. It follows that if this vector is subtracted from the original output vector, x , the difference is another vector which represents the total sectoral outputs created by sector k 's purchasing activities in the rest of the economy:

$$\bar{x}_k = x - x_k.$$

Regard an individual element of vector $\bar{x}_k$, say element i . This is a measure of the total output from sector i created by the input linkage between i and k . As such, it represents the total flow from i to k , not just that part associated with the final demand for goods k .

For convenience, the sum of total outputs of the entire system, t , may be represented as:

$$t = t_k + \bar{t}_k,$$

where t and t_k are the sums of vectors x and x_k , respectively. The difference, $\bar{t}_k$, is that portion of total output attributable to internal purchases by sector k . If sector k externalizes all its inputs, then total system output will equal that of the reduced system:

$$t = t_k.$$

If the above operations are done for every column of the original A matrix, a set of vectors is obtained which can be combined into a new matrix composed of all the total intermediate flows.⁴ We call this the total flow matrix, Z^{total} :

$$Z^{\text{total}} = [z^{\text{total}}_{ij}] = [x_1 \dots x_j \dots x_n].$$

Each element of this matrix, z^{total}_{ij} , expresses the true "total intermediate flow" from sector i to sector j , that is, everything that leaves sector i and sooner or later reaches j . The total i - j flow evidently includes the portion of output from i that goes directly to j (z_{ij}) and, at the same time, the portion which after having left i circulates somewhere in the system and exits through j (z^{exit}_{ij}). The total flow matrix can be used by means of appropriate operations to calculate two additional flow matrices, the "indirect flow" and the "transit flow" matrix, Z^{ind} and Z^{trans} , respectively. These are:

$$Z^{\text{ind}} = Z^{\text{total}} - Z,$$

$$Z^{\text{trans}} = Z^{\text{total}} - Z^{\text{exit}}.$$

The Z^{ind} matrix gives a measure of the indirect i - k linkage, while the Z^{trans} matrix shows how much of the total i - k flow fails to exit through sector k . Note that from the above definitions, the total flow matrix, Z^{total} , can be expressed interchangeably, as the sum of two pairs of matrices, or:

$$Z^{\text{total}} = Z + Z^{\text{ind}} = Z^{\text{exit}} + Z^{\text{trans}}.$$

These matrices characterize the internal structure of input-output systems and verify, to a certain degree, the usefulness of Flow Analysis. For instance, relatively large indirect flows seem to indicate a higher complexity of the system and thus the need for Flow Analysis to recognize them. In order to detect the relative importance of direct flow, we can calculate the

fraction of total flow which is direct, for each nonzero model cell:

$D = [d_{ij}] = Z/Z^{\text{total}} = [z_{ij}/z^{\text{total}}_{ij}]$,
 $i, j = 1, \dots, n$; $z^{\text{total}}_{ij} \neq 0$. This matrix shows the relative sizes of direct flow in the overall system. Its elements may be interpreted, then, as numerical measures of "significance" of direct flows.

An analogous ratio might be calculated for exit flows. It can be shown that the ratio between elements of Z^{exit} and Z^{total} is constant across columns and equal to the exit coefficient, or:

$$e_j = z^{\text{exit}}_{1j}/z^{\text{total}}_{1j} = z^{\text{exit}}_{2j}/z^{\text{total}}_{2j} = \dots$$

$$= z^{\text{exit}}_{nj}/z^{\text{total}}_{nj},$$
 $i, j = 1, \dots, n$; $z^{\text{total}}_{ij} \neq 0$. The exit coefficient vector is:

$$e = [e_1 \dots e_j \dots e_n].$$

This vector indicates the amount of total flow which is actually captured by the Leontief inverse. If e_j is small, Input-Output Analysis overlooks much of the true intermediate flow volume, whereas if e_j is close to one, this is not the case, at least with respect to sector j .

Yet another matrix may be easily defined to measure the structural significance of individual linkages.⁵ Set:

$$\bar{A}_k = [\bar{a}_{ik}], \text{ where}$$

$$\bar{a}_{ik} = a_{ij} \text{ for } k=j, \text{ and } \bar{a}_{ik} = 0, \text{ otherwise;}$$

$$z_k = [z_i], \text{ where } z_i = z_{ij}, \text{ if } k=j.$$

As indicated, $\bar{A}_k$ is a matrix whose elements are zeroes except for those in column k , which is taken as the k -th column in the matrix of technical coefficients, A . z_k is the k -th column of the direct-flow matrix, Z . Now,

$$A = \bar{A}_k + A_k.$$

Therefore, from the well known equation, $x = Ax + y$,

$$x = A_k x + \bar{A}_k x + y, \text{ where}$$

$$\bar{A}_k x = z_k.$$

Further manipulation yields

$$\underbrace{i'x}_t = \underbrace{i'(I - A_k)^{-1}z_k}_{t_k} + \underbrace{i'(I - A_k)^{-1}y}_{t_k}.$$

The scalar, t_k , is the product of the vector of column sums of the matrix $(I - A_k)^{-1}$ and the vector, z_k . Recall that t_k may be interpreted as a measure of the portion of total system output created by the purchasing activities of sector k . Thus, in the following equation, each entry of the row vector, f'_k , is a weight of the contribution of one unit of direct flow, z_{ij} , to the total system output, t . This may be regarded as a sensitivity value of total output to the individual flow.

$$f'_k = [f_{ik}] = i'(I - A_k)^{-1}; i, k = 1, \dots, n.$$

Such a vector may be calculated for each column k , and all vectors so formed, when transposed into column vectors, can be combined into a matrix F :

$$F = [f_{ij}] = [f_1 \dots f_j \dots f_n].$$

Each element in this matrix indicates how important the direct transfer from i to j is to the whole system.

It is possible, as well, to define several indices which measure sectoral significance. One such measure is simply:

$$s_k = \bar{t}_k/t.$$

By the definition of the Z^{total} matrix, this index provides a measure of the total system activity made possible by the purchasing activities of sector k . If all purchases of sector k are externalized, that is, removed from the system entirely and undertaken elsewhere through trade, then the total output of the system will decline by the index s_k , regarded as a fraction of t .

One can also produce various "internal" multipliers by calculating how total system output (income, employment, etc) will be influenced in response to an "externalization" or "internalization" of intermediate inputs to sector k . It can be verified that these multipliers may differ significantly from the classical or "external" multipliers.⁶ A more detailed discussion of this question remains beyond the scope of this article.

Application

In the application, we attempt to analyze the U.S., partitioned into a system of nine regions. The analytical approach derives from several of the concepts just discussed. In particular, we will try to constitute the interregional system in terms of direct, indirect, transit, and exit flows. With our total-flow description and decomposition, we seek to answer several important questions, namely:

- What are the magnitudes of these various flows?
- How important are they to the entire input-output structure?
- Which of these flows are overlooked by Input-Output Analysis, and to what extent?
- How sensitive is the system to changes in intermediate direct flows?
- Are the purchasing activities of certain regions more important to the overall system than others?

The data analyzed are given by Polenske (1980), who published a multiregional model for the U.S. economy in 1963, together with projections for 1980. These data were aggregated by Polanske into nine regions, each of which comprises a number of economic sectors, 19 for 1963, and 18 for 1980. We have further aggregated the economic data for each region, thereby producing two 9×9 transaction tables, one for 1963 (Z_{63}), and the other for the 1980 projections (Z_{80}). (Regional classification is given by Table 1.) We have also calculated the

corresponding total output vectors (x_{63} and x_{80}) and final demand vectors (y_{63} and y_{80}).

Because the 1980 data constitute a projection and not a set of actual measures, we have focused our analysis on the 1963 data. We have taken the liberty, however, to make comparisons between years in order to point out structural changes anticipated by Flow Analysis. These comparisons are to be considered in the light of uncertain data. The 1963 data are old, and those from 1980 are projected from the earlier year, which means that old errors are compounded. Further, it should be emphasized that the analysis undertaken is interregional, and not intersectoral. The relevant coefficients, then, are not technical but trade coefficients, which are often thought to be less stable than their technical counterparts (Richardson, 1972).

TABLE 1
Regional Classification

State			State		
Region*	Number	Name	Region*	Number	Name
1	6	Connecticut	6	1	Alabama
	18	Maine		16	Kentucky
	20	Massachusetts		23	Mississippi
	28	New Hampshire		41	Tennessee
	38	Rhode Island	7	3	Arkansas
2	44	Vermont		17	Louisiana
	29	New Jersey		35	Oklahoma
	31	New York	8	42	Texas
	37	Pennsylvania		2	Arizona
3	12	Illinois	9	5	Colorado
	13	Indiana		11	Idaho
	21	Michigan		25	Montana
	34	Ohio		27	Nevada
	48	Wisconsin		30	New Mexico
4	14	Iowa	9	43	Utah
	15	Kansas		49	Wyoming
	22	Minnesota		4	California
	24	Missouri		36	Oregon
	26	Nebraska		46	Washington
	33	North Dakota		50	Alaska
5	40	South Dakota		51	Hawaii
	7	Delaware			
	8	District of Columbia			
	9	Florida			
	10	Georgia			
	19	Maryland			
	32	North Carolina			
	39	South Carolina			
	45	Virginia			
	47	West Virginia			

*The names of the 9 census regions are:

1 New England	6 East South Central
2 Middle Atlantic	7 West South Central
3 East North Central	8 Mountain
4 West North Central	9 Pacific
5 South Atlantic	

Results

In Table 2, the direct interregional flows for 1963 are presented. It can be seen immediately that the national interregional system is not well connected. More than two thirds of all intermediate transfers are intraregional flows (the on-diagonal cells of the matrix). The largest interregional flows are those between the system's two biggest producers, which happen to be neighbors. These are the East North Central (3) and the Middle Atlantic (2). In general, the overall structure of direct transfers appears homogeneously distributed. There are no zero entries, and the off-diagonal input coefficients range, when calculated, between 0.003 and 0.114.

We display in Table 3 the fraction of total flow which is direct flow for each i - j connection ($Z_{63}/Z_{63}^{\text{total}}$). Like the Z_{63} matrix, the diagonal elements here are also large. Little that originates from some sector i and circulates through the system returns to i . The off-diagonal elements tell us to what extent trade relationships are direct manifestations of exchange. Since these are relatively small throughout the table, we are left with the impression that indirect flows between regions are important. This result points to complexity in trade relations.

Total intermediate flow can be partitioned into direct and indirect flows as well as into exit and transit flows. Input-Output Analysis provides a measure of exit flow or the amount of intermediate output from some component i necessary to satisfy final demand for the output

TABLE 2
US Interregional:Direct Flow—1963 (Z_{83})
(Billions of 1963 Dollars)

	1	2	3	4	5	6	7	8	9	TOTAL
	37.0	11.0	4.5	1.4	2.9	.6	1.0	.5	1.7	60.6
2	12.8	147.7	23.7	4.6	15.5	3.7	4.8	1.6	6.8	221.2
3	4.4	22.9	164.1	14.7	12.7	6.9	5.6	2.1	8.8	242.2
4	.9	3.8	12.7	53.1	2.1	2.8	6.3	2.8	3.0	87.5
5	2.6	15.6	8.1	2.7	82.6	6.3	3.3	.8	3.8	125.8
6	.7	2.9	4.8	2.0	7.4	25.4	4.2	.5	1.2	49.1
7	1.2	4.3	4.7	4.4	4.6	3.1	61.4	2.8	2.7	89.2
8	.3	.5	1.4	3.0	.4	.2	2.4	22.5	5.5	36.2
9	1.4	4.0	3.9	1.9	2.1	.8	2.1	6.0	114.3	136.5
TOTAL	61.3	212.7	227.9	87.8	130.3	49.8	91.1	39.6	147.8	1048.3
FINAL DEMAND—1963 (y_{83})										
	1	2	3	4	5	6	7	8	9	TOTAL
	36.1	120.1	114.2	45.3	73.3	27.5	50.5	23.3	90.3	580.6
TOTAL OUTPUT—1963 (x_{83})										
	1	2	3	4	5	6	7	8	9	TOTAL
	96.7	341.3	356.4	132.8	199.1	76.6	139.7	59.5	226.8	1628.9

of component j . But Input-Output Analysis does not measure the transit flows which provide additional insight into the structural importance of individual components, or regions, as the case may be. Recall from the theoretical discussion that the ratio of transit to exit flow is fixed for each sector. The fourth row in Table 8 gives a set of exit flow/total flow ratios for the 9 region 1963 economy. These represent the portions of total flow accounted for by exit flow in each region. The most significant "implicit" role in the interregional structure is played by the East South Central Region (6), for which 45 per cent of total output is actually transit flow. The Pacific Region (9) plays a relatively minor

role, with only 18 per cent of total flow accounted for by transit. Thus, Input-Output Analysis, as a rule, tends to underestimate the true volume of interconnecting flows.

Table 4 provides "structural importance values" for the various direct, interregional flows. Each entry in the table is a multiplier which gives the total system output generated per unit of direct flow (F_{83}). As can be seen, the two most important direct transfers occur between the Mountain Region (8) and two others, New England (1) and East South Central (6). For each of these connections, a unit of direct flow accounts for more than 2.85 units of total system output, t . In general, flows between geo-

TABLE 3
US Interregional:Direct Flow/Total Output—1963 (D_{83})

	1	2	3	4	5	6	7	8	9	TOTAL
1	.967	.541	.394	.324	.370	.235	.289	.288	.346	3.753
2	.488	.934	.460	.289	.431	.304	.324	.257	.354	3.841
3	.311	.425	.941	.444	.373	.376	.304	.257	.367	3.797
4	.251	.308	.509	.955	.253	.402	.481	.451	.348	3.957
5	.333	.484	.373	.307	.950	.477	.346	.240	.361	3.871
6	.291	.345	.458	.401	.565	.954	.537	.291	.313	4.154
7	.303	.343	.347	.422	.392	.418	.972	.447	.340	3.984
8	.278	.189	.326	.527	.205	.180	.497	.973	.570	3.744
9	.317	.334	.322	.278	.286	.241	.308	.463	.983	3.534
TOTAL	3.537	3.904	4.131	3.947	3.825	3.587	4.058	3.666	3.982	34.636

TABLE 4
US Interregional:Direct-Output Structural Importance—1963 (F_{63})

	1	2	3	4	5	6	7	8	9	TOTAL
1	1.000	2.333	2.524	2.677	2.605	2.695	2.676	2.723	2.676	21.911
2	2.589	1.000	2.439	2.632	2.523	2.651	2.637	2.689	2.647	21.808
3	2.715	2.522	1.000	2.631	2.657	2.712	2.706	2.754	2.721	22.419
4	2.828	2.698	2.497	1.000	2.768	2.809	2.749	2.800	2.807	22.956
5	2.773	2.553	2.570	2.766	1.000	2.707	2.741	2.827	2.788	22.725
6	2.782	2.612	2.501	2.683	2.561	1.000	2.685	2.810	2.776	22.409
7	2.807	2.677	2.635	2.667	2.725	2.732	1.000	2.780	2.774	22.797
8	2.867	2.760	2.713	2.730	2.822	2.860	2.742	1.000	2.595	23.089
9	2.806	2.694	2.656	2.768	2.754	2.812	2.780	2.751	1.000	23.020
TOTAL	23.167	21.852	21.534	22.553	22.415	22.979	22.715	23.133	22.784	203.133

graphically distant regions seem to be most important to the total system processing.

In Tables 5 through 8, comparisons over time are made using the 1980 projections in addition to the 1963 data. A general impression that rises immediately from an examination of these tables is that interregional linkages have tended to develop unevenly over time. The Mountain Region (8) and New England (1) have emerged as important purchasers of commodities from other regions, as can be seen in Table 5, which presents the ratios of 1980 direct flow to that of 1963. The West South Central (6) and South Atlantic (5), in turn, were the quickest in

growing as suppliers of commodities for other regions.

Since 1963, the importance of direct linkages has risen. This can be seen from Table 6, which shows how the ratios of direct flow in 1980 to those of 1963 have changed. The Mountain Region (8) in particular, has evolved to a total flow pattern largely composed of indirect flows. If increasing indirect linkage may be regarded as a sign of persistent integration, then Table 6 indicates that the U.S. has continued to develop as an economic unit.

In Table 8 the index s represents the percentage that total output of the interregional

TABLE 5
US Interregional:Direct Flow—1980/1963 (Z_{80}/Z_{63})
(Billions of 1963 Dollars)

	1	2	3	4	5	6	7	8	9	TOTAL
1	3.67	1.17	1.00	5.79	1.03	1.50	.70	1.20	.47	16.53
2	3.48	1.00	.90	3.09	1.77	1.27	.67	1.25	1.00	14.42
3	3.14	1.33	.71	3.41	1.51	7.68	1.64	2.43	.73	22.57
4	3.56	.92	.68	1.59	1.57	2.61	3.44	3.54	1.30	19.20
5	4.73	1.55	.80	2.48	1.22	3.62	4.12	35.38	1.74	55.63
6	5.00	2.00	.85	2.35	1.34	5.73	5.48	2.40	.75	25.90
7	25.50	5.91	3.00	2.75	2.59	6.35	2.30	7.39	1.52	57.31
8	4.00	1.60	2.86	2.00	3.25	4.50	3.17	12.92	4.62	38.91
9	3.50	.63	.51	2.68	1.29	1.63	2.05	1.55	2.14	15.97
TOTAL	56.57	16.10	11.31	26.14	15.57	34.89	23.56	68.05	14.26	266.45
FINAL DEMAND—1980/1963 (y_{80}/y_{63})										
	1	2	3	4	5	6	7	8	9	TOTAL
	1.93	2.00	2.06	1.93	2.40	2.31	2.37	2.21	2.17	19.39
TOTAL OUTPUT—1980/1963 (x_{80}/x_{63})										
	1	2	3	4	5	6	7	8	9	TOTAL
	2.45	1.50	1.51	1.76	1.99	3.42	2.86	6.54	2.09	24.13

TABLE 6
US Interregional:Direct Flow/Total Output—1980/1963 (D_{80}/D_{63})

	1	2	3	4	5	6	7	8	9	TOTAL
1	1.012	.705	.756	1.073	.678	.537	.303	.385	.439	5.888
2	1.303	.980	1.203	1.451	1.377	.828	.561	.614	1.155	9.471
3	1.232	1.265	.956	1.478	1.262	1.803	.716	.941	.898	10.551
4	.918	.770	.882	.984	1.133	.993	1.124	1.040	.830	8.675
5	1.277	1.153	.923	1.022	.972	1.278	1.194	2.686	.790	11.295
6	.587	.657	.519	.600	.599	1.016	.749	.336	.345	5.407
7	1.669	1.327	1.188	.814	.966	1.154	.944	1.149	.571	9.782
8	.272	.296	.532	.345	.499	.386	.384	1.017	.427	4.158
9	1.063	.709	.739	1.188	1.042	.803	.956	.896	1.003	8.399
TOTAL	9.333	7.860	7.697	8.956	8.528	8.798	6.932	9.064	6.457	73.625

system would drop if region j externalized all its purchases. It is interesting to note that the Mountain Region (8) shows the lowest percentage in 1963, but the highest one in 1980. Presently, if this region externalized all its purchases to some foreign country, the total national output would drop by approximately 14 percent, a surprising amount. In purely quantitative terms, the Mountain Region has evolved from a position of relative unimportance, vis-a-vis the interregional system, to one of prominence. It is also interesting to observe the change which has occurred in the East North Central Region (3). In 1963, this region was most important to the overall system, while according to the 1980 projections, it is least important, while sharing this position with the South Atlantic Region (5). This change, in the context of that observed for the Mountain Region (8), is suggestive of the vast dislocations which have occurred in the past few decades due to the migration of people and capital to the so-called sunbelt. It also shows how the western

part of the sunbelt has been the site of greater change than its eastern counterpart, the South Atlantic Region (5), despite the gains enjoyed by a fast growing state such as Florida.

In Table 7 are presented the changes in "structural importance" of particular direct interregional flows (F_{80}/F_{63}). We observe declining importance of intermediate shipments from such regions as East North Central (3), West North Central (4), and Middle Atlantic (2), while at the same time new regions are becoming more important to the overall structure, notably, the Mountain Region (8), and New England (1).

It should be pointed out that, although the Mountain Region (8), as a whole, contributed little to total output in 1963, each unit of its intermediate output was nevertheless very important (Table 4). Furthermore, this region experienced the greatest increment, across all regions, in contribution to total flow per unit (Table 7). In both total and per unit terms, then, the Mountain Region (8) has evolved to a posi-

TABLE 7
US Interregional:Direct-Output Structural Importance—1980/1963 (F_{80}/F_{63})

	1	2	3	4	5	6	7	8	9	TOTAL
1	1.000	1.923	1.926	1.814	1.859	1.771	1.660	1.789	1.840	15.581
2	.781	1.000	.858	.806	.828	.785	.767	.793	.814	7.431
3	.618	.659	1.000	.635	.641	.619	.611	.607	.632	6.022
4	1.114	1.198	1.251	1.000	1.196	1.150	1.154	1.123	1.182	10.368
5	.713	.750	.765	.718	1.000	.703	.706	.705	.723	6.783
6	1.510	1.597	1.565	1.515	1.584	1.000	1.467	1.484	1.531	13.254
7	.925	.969	.970	.903	.936	.851	1.000	.883	.935	8.373
8	2.037	2.112	2.143	2.054	1.980	2.012	2.009	1.000	2.199	17.546
9	1.072	1.109	1.129	1.069	1.078	1.065	1.062	.903	1.000	9.487
TOTAL	9.770	11.318	11.607	10.514	11.102	9.955	10.436	9.286	10.857	94.844

TABLE 8
US Interregional: Sectoral Structure Indices—1963, 1980, 1980/1963

	1	2	3	4	5	6	7	8	9	TOTAL
S_{63}	.063	.192	.199	.089	.127	.056	.087	.043	.126	.982
S_{80}	.116	.121	.088	.101	.088	.108	.118	.137	.134	1.010
S_{80}/S_{63}	1.852	.629	.442	1.130	.689	1.911	1.354	3.208	1.063	12.279
e_{63}	.618	.656	.627	.587	.653	.550	.660	.640	.817	5.808
e_{80}	.709	.685	.575	.610	.609	.567	.487	.538	.873	5.653
e_{80}/e_{63}	1.148	1.045	.916	1.039	.933	1.031	.739	.839	1.069	8.759

tion of special importance to the interregional system.

Conclusion

The total flow concept provides for a decomposition and reconstruction of input-output data which leads to useful insights, not directly discernible in the Leontief inverse. In particular, by introducing measures of structural significance, total flow enables the analyst to assess both system change and response to special, important impacts. It should be evident from the application, that the total flow approach has provided a means to describe system evolution in terms of the changing importance of various flows and linkages.

What is perhaps most striking is how the method captures the recent history of the sunbelt phenomenon. Recent migration to the South and West has fundamentally altered the spatial economy of the U.S. It has been documented that in many categories, including, for example, numbers of new jobs, the Mountain Region (8) is leading the nation (Weinstein & Firestone, 1978). At the same time, the East (3) and West North Central (4) Regions are in marked decline. The method presented here indicates quite clearly such a process of change. It is interesting, too, that the results show a vital and resilient New England (1). As it turns out, New England is the one northern region which has been able to avoid the more serious consequences of sunbelt migration (Jackson et al., 1981).

One of the possible explanations for the pre-eminence of the Mountain Region resides in the concept of an economic center, once unquestionably anchored in the Northeast. As people and income have migrated south and west, the center too has drifted in the same direction. Perhaps this drift has reached an equilibrium point

whereby the geographic and economic center are not coincident. This spatial phenomenon may have provided the Mountain Region with locational advantages it once did not possess.

Another interesting result is that of the rising importance of the East and West South Central Regions (6 and 7). In recent years, total shipping in the Gulf States has exceeded that of any part in the Northeast (Vining et al, 1982). Furthermore, railroad freight traffic in the south has grown steadily in the postwar era, a trend matched by steady decline in the Northeast. Apparently, the South is not the isolated region it once was; transit flows and trade in general now figure as significant components in the overall economic processing.

Perhaps the most potentially useful application of the total flow concept was not developed in this paper and awaits future treatment. With rapidly integrating world markets, intranational flows are more vulnerable to the threat of externalization than ever before. A total flow concept may help to assess the national consequences of participation in world markets.

FOOTNOTES

¹Recently, Loviscek (1982) investigated difference between demand- and supply-oriented input-output models and indicated substantial differences between both approaches. See also Augustinovic (1970) and DaSilva (1981).

²The meaning of the Leontief inverse is often misunderstood. For example, Blin & Murphy (1974, p. 438) claim that the elements of the Leontief inverse represent "... the total amount of input dependence of sector j on the output of sector i ...". This belief leads them to the construction of the "interrelatedness" measure which does not correctly express the internal interindustry connections. In order to take into account all intermediate relationships in input-output models one has to consider not final demand inputs but other interindustry flows as well.

In ecology, Hannon (1973, p. 536) calls the matrix Z^{dest} (matrix E in Hannon's notation) the total flow matrix which represents the sum of direct flows which is "... actual energy flow through a component ..." and indirect flows "... necessary in order that the direct energy flow could

occur through a given component . . .” Actually, flow in excess of these indirect flows is necessary to secure the occurrence of those direct flows.

³The idea of flow externalization is borrowed from Ulanowicz (1980) who in his seminal article examines the impacts of changes in flow structure.

⁴It is worthwhile to note that the total flow algorithm presented here is only one of several possible methods. It is not a particularly convenient, especially when dealing with large matrices. By appealing to specific properties of the Leontief inverse (West, 1982) or, by using the product form of the inverse (Hadley, 1961), one can calculate total flow matrix directly from the “old” inverse $-(I - A)^{-1}$. Moreover, total flow matrix can be expressed directly as a function of the elements of old inverse and final demand, y .

⁵Structural importance of i - j links in input-output models has been dealt within so-called Sensitivity Analysis, which has been recently reconsidered in both economics (West, 1982) and ecology (Bosserman, 1981). Bosserman calculates sensitivity values vis-a-vis the changes in the Leontief matrix, $(I - A)$, West - with a respect to the technical coefficient matrix, A . By contrast, in our approach, sensitivity is related to the changes in the direct flow matrix, Z .

⁶It should be noted that all the internal multipliers suffer from a common deficiency, namely, they are nonlinear. For instance, each subsequent unit of externalized i - j direct flow causes the uneven changes in total system output. Therefore, one can deal only with average numbers which are less appealing than the constants, as provided by standard linear “external” multipliers. This weakness, however, does not undermine the significance of internal multipliers.

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