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Spatial Structure and Spatial Interaction: 25 Years Later

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Abstract

In the 1970s, spatial autocorrelation (i.e., local distance and configuration effects) and distance decay (i.e., global distance effects) were suspected of being intermingled in spatial interaction model specifications. This convolution was first treated in a theoretical context by Curry (1972), with some subsequent debate (e.g., Curry, Griffith, and Sheppard 1975). This work was followed by a documentation of the convolution (e.g., Griffith and Jones 1980) and further theoretical treatment of the role spatial autocorrelation plays in spatial interaction modeling (e.g., Griffith 1982). But methodology did not exist at the time—or even soon thereafter—to easily or fully address spatial autocorrelation effects within spatial interaction model specifications, a contention attested to and demonstrated by the cumbersome and difficult-to-implement techniques employed by, for example, Bolduc, Laferrière, and Santarossa (1992, 1995) and Bolduc, Fortin, and Gordon (1997). Today, however, eigenfunction-based spatial filtering offers a methodology that can account for spatial autocorrelation effects within a spatial interaction model. This paper updates work from the early 1980s, extending it with spatial filtering methods.

Keywords: Distance decay; Gravity model; Spatial autocorrelation; Spatial filter; Spatial interaction

JEL classification: C21; J20; R49

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1. INTRODUCTION AND BACKGROUND

The (unconstrained) gravity model of spatial interaction describes flows (F_{ij}) between origin location i and destination location j with the following equation (see Sen and Smith 1995):

$$(1) \quad F_{ij} = \kappa O_i^\alpha D_j^\beta e^{-\gamma d_{ij}} + \varepsilon_{ij},$$

where O_i denotes a mass term (e.g., population) for origin i ,
 D_j denotes a mass term for destination j ,
 d_{ij} denotes the distance separating locations i and j ,
 ε_{ij} denotes the random noise for the flow between locations i and j , and
 κ , α , β , and γ are parameters.

Wilson (1970) establishes the exponential specification of the distance function. Meanwhile, the four parameters may be interpreted as follows.

κ is a constant of proportionality that rescales the product of two quantities,
 α is an exponent that inflates a too-small O_i and deflates a too-large O_i ,
 β is an exponent that inflates a too-small D_i and deflates a too-large D_i , and
 γ is a friction of distance measure that increases as distance becomes increasingly restrictive to the movement of flows.

With only four parameters estimated from $n(n - 1)$ flows—within areal unit F_{ii} flows almost always are set aside—equation (1) furnishes a surprisingly good description of a disparate set of flows across a wide range of geographic landscapes.

One weakness of equation (1) is that it overlooks spatial autocorrelation contained in its origin and its destination geographic distributions. Based upon the spatial linear operator commonly employed today in spatial statistics and spatial econometrics, Griffith and Jones (1980, p. 188) argue that, using matrix notation, a specification involving the following was needed:

$$(2) \quad (\mathbf{I} - \rho_o \mathbf{W})\mathbf{F} \text{ and } \mathbf{F}(\mathbf{I} - \rho_d \mathbf{W}),$$

where $\mathbf{I}$ is an n -by- n identity matrix; $\mathbf{F}$ is the n -by- n flows matrix (with its diagonal coded with a missing value); $\mathbf{W}$ is a row-standardized geographic connectivity matrix (i.e., based upon a binary 0-1 connectivity matrix $\mathbf{C}$ for which $c_{ij} = 1$ if areal units i and j are neighbors, and 0 otherwise); and ρ_o and ρ_d , respectively, are the spatial autocorrelation parameters for the origin and the destination geographic distributions. Concerned about this problem, Griffith and Jones (1980) explore the relationship between spatial structure and spatial interaction at the intra-urban level by examining journey-to-work data for 24 selected Canadian cities. Their findings conclude that distance-decay exponents are

strongly influenced by geographic structure and the geometry of origins and destinations. They consistently uncover statistically significant, positive spatial autocorrelation in gravity model parameters. But computing power in the late 1970s and early 1980s was insufficient to estimate their proposed model specification, which involves an n^2 -by- n^2 matrix. More recently, however, LeSage and Pace (2005) outline feasible methodology for estimating this revised equation.

The purpose of this paper is to revisit the spatial-structure spatial-interaction problem by expanding the LeSage-Pace formulation to include a spatial filtering specification. Results are applied to German employment flows data.

2. SPATIAL FILTERING: AN OVERVIEW

One of the difficulties associated with spatial statistics revolves around its implementation. One of the attempts to address this problem has fostered the development of non- and semi-parametric spatial filtering techniques. These furnish an approach to dealing with spatial autocorrelation in regression analysis by separating a variable's spatial and aspatial effects, allowing spatial statistical analysts to use conventional regression models for their data analyses. Four types of spatial filters exist for georeferenced data analysis, namely spatial linear operators (e.g., Griffith 1979), Getis's G_i specification (e.g., Getis and Griffith 2002), Griffith's eigenfunction specification, and the Legendre-group's PCNM specification (e.g., Griffith and Peres-Neto 2006). Not surprisingly, all four of these versions are variants of the same basic conceptualization.

The Griffith specification, which continues to be developed by Tiefelsdorf and Griffith (2007), is a transformation procedure that depends on mathematical expressions, known as eigenfunctions, that characterize the aforementioned geographic connectivity matrix C . This transformed connectivity matrix appears in the numerator of the Moran Coefficient (MC) spatial autocorrelation test statistic. This decomposition rests on the following property: when mapped spatially, the first eigenvector, say E_1 , is the set of real numbers that has the largest MC value achievable by any set of real numbers for the spatial arrangement defined by the geographic connectivity matrix C ; the second eigenvector is the set of real numbers that has the largest achievable MC value by any set that is uncorrelated with E_1 ; the third eigenvector is the third such set of values; and so on through E_n , the set of real numbers that has the largest negative MC value achievable by any set that is uncorrelated with the preceding $(n - 1)$ eigenvectors. As such, these eigenvectors furnish distinct map pattern descriptions of latent spatial autocorrelation in georeferenced variables, because they are both orthogonal and uncorrelated. Their corresponding eigenvalues index the nature and degree of spatial autocorrelation portrayed by each eigenvector.

The spatial filter is constructed from some linear combination of a subset of these eigenvectors. A stepwise regression procedure can be used to select those eigenvectors that account for the spatial autocorrelation in a response variable. This stepwise selection

can be based upon the conventional residual sum-of-squares-minimization criterion or, as proposed by Tiefelsdorf and Griffith (2007), a residual MC-minimization criterion.

3. SPATIAL AUTOCORRELATION IN ORIGIN AND DESTINATION ATTRIBUTE VARIABLES

Daily German home-to-work trip commuting flows data for the single year of 2002 and the geographic resolution of NUTS-3 (the 439 German kreises) constitute the empirical basis of analyses summarized here (see Patuelli et al. 2006). These data were collected by the (German) Federal Employment Services (Bundesanstalt für Arbeit, BA). Each origin mass term was calculated by summing all entries in its corresponding row of the flows matrix. The geographic distributions of these mass terms—namely, the logarithms of the numbers of employees and of jobs—are portrayed in Figure 1. Each destination mass term was calculated by summing all entries in its corresponding column of the flows matrix. Once these origin and destination terms were computed, attention was restricted to the $n(n - 1) = 192,282$ off-diagonal values in the flows matrix. The total number of workers geographically distributed across the origins/destinations is 27,454,023; the total number of inter-kreise flows is 9,685,399. Separation was measured as the distance in thousands of kilometers between kreise centroids (computed in ArcGIS). Poisson regression estimation of equation (1) with and without the diagonal entries yields the following.

Parameter	With Diagonal Flows	Without Diagonal Flows
$\hat{\kappa}$	0.0372	0.0001
$\hat{\alpha}$	0.4740	0.5262
$\hat{\beta}$	0.7295	1.0010
$\hat{\gamma}$	3.4787	1.6837
Pseudo- R^2	0.9188	0.5067

Substantial overdispersion is present in these data (deviance statistic = 201 and 102, respectively) that cannot be accounted for by replacing the Poisson assumption with a negative binomial assumption. (The deviance statistic decreases to roughly 1, but the pseudo- R^2 , for example, plummets to approximately 0.5 and 0.2, respectively). This situation suggests the presence of spatial autocorrelation complications in these data that need to be addressed by accounting for this spatial autocorrelation.

Parameter estimates for this unconstrained gravity model suggest that the size of labor pools for the origins are too large and need to be deflated (i.e., $\hat{\alpha} < 1$), whereas the number of employment opportunities at the destinations are appropriate for the observed inter-kreise journey-to-work flows (i.e., $\hat{\beta} = 1$). Not surprisingly, the distance decay parameter (i.e., $-\hat{\gamma} < 0$) suggests strong deterrence by the separation of place of residence and place of work.

FIGURE 1. (A): the geographic distribution of workers based upon the logarithm of flow matrix origin totals. (B): the geographic distribution of jobs based upon the logarithm of flow matrix destination totals. Note: grayscale darkness is directly proportional to the magnitude of the logarithms.

4. SPATIAL AUTOCORRELATION IN STOCKS: MAP PATTERNS IN ORIGIN AND DESTINATION FLOW TOTALS

In keeping with equation (2), the following two pure spatial statistical simultaneous autoregressive (SAR) models were estimated for the logarithmically (LN) transformed origin and destination employee totals:

$$LN(O_i) = (1 - \rho)\mu + \rho \sum_{j=1}^n LN(O_j) \quad \text{and} \quad LN(D_i) = (1 - \rho)\mu + \rho \sum_{j=1}^n LN(D_j).$$

Results from this exercise may be summarized as follows:

<u>Feature</u>	<u>MC</u>	<u>$\hat{\rho}$</u>	<u>pseudo-R²</u>	<u>P(S–W)</u>
Origin	0.41	0.54	0.269	0.001
Destination	0.40	0.55	0.296	0.011

where P(S–W) denotes the probability of the Shapiro-Wilk normality diagnostic statistic under the null hypothesis of normality. These results indicate that moderate positive spatial autocorrelation is displayed by the geographic distributions of employee residences and employment. Roughly a quarter of the geographic variance of these geographic distributions is accounted for by tendencies for employee residences, as well as for employment, to concentrate in nearby kreises. This description is also somewhat compromised by deviations from a bell-shaped curve. This spatial autocorrelation could

partly account for the overdispersion previously detected in the Poisson modeling of the journey-to-work flows.

5. A SPATIAL FILTER POISSON MODEL OF SPATIAL FLOWS

Using matrix notation, the following spatial filter models were estimated with the origin and with the destination data:

$$O = \mu_O 1 + E_k \beta_k + \varepsilon_O \text{ and } D = \mu_D 1 + E_k \beta_k + \varepsilon_D,$$

where μ_j ($j = O, D$) is the mean of the corresponding geographic distribution, ε_j ($j = O, D$) is the error term (i.e., the aspatial variable component) of the corresponding geographic distribution, and $E_k \beta_k$ is the spatial filter constructed from the set of k judiciously selected (i.e., with stepwise regression techniques) eigenvectors extracted from a modified version of the geographic connectivity matrix C . Spatial filters for the geographic distributions of employees and jobs are portrayed in parts (A) and (B) of Figure 2. Estimation results include the following.

<u>Feature</u>	<u>Origins</u>	<u>Destinations</u>
MC for $E_k \beta_k$	0.791	0.811
# eigenvectors	29	26
R^2	0.418	0.434
Residual MC	-0.02	-0.05

Of 439 possible eigenvectors, 98 have an adjusted MC value (i.e., $MC/MC_{\max}$) of at least 0.25 (hence accounting for at least about 5 percent of the geographic variance). Roughly a third of these vectors were selected by stepwise regression to construct each spatial filter. These linear combinations of eigenvectors, which represent strong positive spatial autocorrelation, account for approximate 40 percent of the variance in each geographic distribution and account for all but a trace amount of spatial autocorrelation.

The selected eigenvectors can be properly linked to the origin and destination data with the Kronecker product matrix operation (see LeSage and Pace 2005), generating the following Poisson regression results.

<u>Parameter</u>	<u>Original Data</u>	<u>Spatially Filtered Data</u>	<u>Decomposed Data</u>
$\hat{\alpha}$	0.53	0.88	0.80
$\hat{\beta}$	1.00	1.27	1.10
$\hat{\gamma}$	1.68	1.83	1.75
Deviance	101.7	108.1	92.8
Pseudo- R^2	0.507	0.493	0.570
# eigenvectors	0	0	59

FIGURE 2. (A): the spatial filter for the geographic distribution of workers based upon flow matrix origin totals. (B): the spatial filter for the geographic distribution of jobs based upon flow matrix destination totals. (C): the spatial filter for the geographic distribution of workers based upon inter-kreise flows. (D): the spatial filter for the geographic distribution of jobs based upon inter-kreise flows. Note: grayscale darkness is directly proportional to spatial filter values.

These findings imply that: a small amount of the overdispersion is linked to the spatial filter map patterns; the origin mass term is too large; the destination mass term is too small; roughly 50 percent of the variation in inter-kreise flows is described by the gravity model; and distance deterrence is somewhat greater than is indicated by the raw data analysis. This interaction between spatial autocorrelation and distance decay is the effect identified and addressed by Curry (1972). Again, a negative binomial specification fails to adequately account for the overdispersion here, suggesting that a quasi-likelihood

estimation procedure should be employed, rendering a scaling factor of roughly 10 and removing only 1 eigenvector from the spatial filter on the basis of statistical significance.

5.1 The Constructed Spatial Filters

The geographic distributions of log-workers and log-jobs as well as their corresponding spatial filters have nearly the same level of positive spatial autocorrelation, respectively, of $MC = 0.40$ and $MC = 0.80$ for each of the two pairs. In both cases the PRESS/MSE ratio (a cross-validation measure) is about 1.02. The origin spatial filter was constructed with 29 whereas the destination spatial filter was constructed with 26 eigenvectors. Of these, 25 eigenvectors are common to both geographic distributions, including the first two, which represent the highest possible levels of positive spatial autocorrelation as well as global map patterns respectively reflecting a north-south and an east-west gradient. Eigenvector E_7 has been removed from the equation based upon quasi-likelihood estimation criteria.

The deconstructed origin and destination variables, rewritten in terms of their associated spatial filters, yield the following gravity model equation:

$$(3) \quad F_{ij} = \kappa e^{\sum_{h=1}^{28} E_{i,h} \beta_{O,h}} O_i^{*\alpha} e^{\sum_{k=1}^{25} E_{i,k} \beta_{D,k}} D_j^{*\beta} e^{-\gamma d_{ij}} + \epsilon_{ij},$$

where $O_i^{*\alpha}$ and $D_j^{*\beta}$ respectively denote the spatially filtered components of the origin

and destination totals, and $e^{\sum_{h=1}^{28} E_{i,h} \beta_{O,h}}$ and $e^{\sum_{k=1}^{25} E_{i,k} \beta_{D,k}}$ respectively denote the origin and destination spatial filters based upon inter-kreise flows. These two spatial filters for the geographic distributions of employees and jobs are portrayed in parts (C) and (D) of Figure 2.

Scatterplots of these flows-based spatial filters as well as their stock-based (i.e., flow matrix row and column totals) counterparts appear in Figure 3. Their comparison reveals that the geographic distributions of jobs and workers tend to reflect a very similar spatial structure, whereas the geographic distributions of workers leaving kreises for jobs and workers entering kreises for jobs have a much less similar spatial pattern. Estimation results here include the following.

<u>Feature</u>	<u>Origins</u>	<u>Destinations</u>
MC for $E_k \beta_k$	0.980	0.898
# vectors	74	61
R^2	0.146	0.065
Residual MC	0.26	0.37

Moreover, the spatial filters include far more eigenvectors, represent a much stronger level of positive spatial autocorrelation, and account for no more than about 40 percent of

FIGURE 3. Scatterplots of the origin and destination spatial filters. (A): spatial filters constructed with the origin and destination log-totals. (B): spatial filters constructed with the inter-kreise flows.

the spatial autocorrelation displayed by the geographic distributions of origin and destination totals.

Eigenvectors included in each of the spatial filters are enumerated in Table 1. The flows-based spatial filters capture a far more complex map pattern than do their stock-based counterparts. Twenty-eight eigenvectors are common to both sets of spatial filters, but only 13 are held in common by all four.

TABLE 1

Eigenvectors Selected for Spatial Filter Construction

unique to workers	common	unique to jobs
<i>based solely on the origin and destination geographic distribution totals</i>		
E₃₁, E₃₄, E₈₉, E₉₁	E₁, E₂, E₇, E₈, E₁₀, E₁₁, E₁₂, E₁₈, E₂₁-E₂₈, E₃₀, E₃₈, E₄₇, E₄₈, E₅₂, E₅₄, E₆₀, E₆₄, E₇₃	E₃₉
<i>Based on inter-kreise flows</i>		
E₃₄, E₁₁, E₂₁-E₂₃, E₂₅, E₄₇, E₅₄, E₆₀	E₁, E₂, E₇, E₈, E₁₀, E₁₂, E₁₈, E₂₄, E₂₆, E₂₇, E₃₀, E₃₈, E₄₈, E₃₁, E₃₉, E₈₉, E₉₁	E₆₄, E₇₃
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E₁₃, E₁₇, E₂₀, E₃₅, E₄₀, E₄₄, E₄₆, E₄₉, E₅₅, E₅₇, E₆₁, E₆₃, E₆₇, E₇₂, E₇₆, E₈₆, E₉₂	E₃, E₄, E₆, E₁₅, E₁₆, E₁₉, E₃₂, E₃₃, E₃₆, E₃₇, E₄₂, E₄₃, E₄₅, E₅₀, E₅₃, E₅₆, E₅₈, E₆₂, E₆₅, E₆₆, E₆₉-E₇₁, E₇₄, E₇₅, E₈₀, E₈₂, E₈₇, E₉₀, E₉₅, E₉₇	E₅, E₉, E₁₄, E₅₁, E₅₉, E₆₈, E₇₇, E₈₃, E₈₄, E₉₆, E₉₈
NOTE: Those eigenvectors below the dotted line are unique to the flows-based spatial filters, while those above the dotted line are common to both the stock- and the flows-based spatial filters.		

6. CONCLUSIONS AND IMPLICATIONS

More than 25 years have past since Griffith and Jones (1980) attempted to address numerically the convolution of spatial autocorrelation and distance decay effects in spatial interaction modeling. Spatial autocorrelation plays an undercover role in spatial interaction. Indeed, evidence gleaned both then and now confirms that spatial autocorrelation (i.e., local distance and configuration effects) and distance decay (i.e., global distance effects) intermingle in spatial interaction model specifications. Spatial filtering allows at least part of this role to be teased out of data. Empirical findings summarized here confirm Curry's (1972) earlier hypothesis that spatial autocorrelation effects are confounded with distance decay effects during the estimation of simple gravity model parameters. The complexity of the spatial filters constructed here suggests that future research should address this problem in terms of network spatial autocorrelation. The tendencies of interest should be of similar nearby flow departures and nearby flow arrivals rather than similar nearby origin and nearby destination mass terms. In addition, comparisons of the spatial filtering approach to this problem with a spatial autoregressive approach, such as the specification put forth by LeSage and Pace (2005), are needed.

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