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Location of Retail Stores in City Center and Outskirts under Spatial Price Competition: Improvements in Radial and Ring Roads[*]

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Abstract: Retail stores locate in both the center and the outskirts of most cities. Their locations can change with transportation improvements. This paper presents a spatial price competition model with central and outlying commercial areas. It illustrates the relationship between retail store location and transportation improvements. An improvement of a radial road connecting the city center and the outskirts contributes to commercial development in the central area. However, an improvement of a ring road in the outskirts does not always engender a decline in the number of stores in central commercial areas.

Keywords: retail stores location, spatial price competition
JEL Codes: L11, L13, R12, R32

1. INTRODUCTION

The location of stores has remained a pivotal issue for regional scientists and economic geographers. This paper examines the location of retail stores scattered in both central and surrounding commercial areas. The central area supplies goods similar to those supplied in the outskirts, but those goods are not completely homogeneous. In the central area, retail stores that constitute the downtown area sell luxury goods and high-quality services. In contrast, shopping malls and specialized stores in the outskirts sell daily-use commodities and services. Retail stores in central areas and outskirts compete by selling imperfect substitutes. Transportation improvements and the expansion of suburban areas change the relative superiority that prevails between the two commercial areas in central and outlying areas. Accordingly, each area may experience increased prosperity or a decline in commerce.

Indeed, over the past several decades, the loss of central city commerce has been brought to the public's attention repeatedly in many developed countries.[1] The so-called "hollowing out"

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[1] For example, in the U.K. up to the early 1980s, most shopping developments occurred in town centers. However, during the Thatcher era of the 1980s, many shopping centers located in the suburbs because of transportation improvements in outlying areas and deregulation of the location of stores. Accordingly, town-center development was less common than out-of-town development. Planning Policy Guidance 6, published in 1996, was intended to reverse the move towards out-of-town development and thereby prevent the replication of the U.S. car-dependent pattern of retail development in the U.K. (See Reynolds and Cuthbertson (2003, Chapter 4) for the transition of the hollowing-out and the revitalization of the urban center in

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of central commerce has been caused by motorization, transportation improvements in outlying areas, and the consequent residential and industrial expansion of suburban areas. As a typical example, construction of a new road in outlying areas promotes the construction and expansion of shopping malls along the new road while simultaneously engendering the hollowing out of central business districts.

Store location has been studied at length, starting from classic theories such as Hotelling's (1929) "ice cream vendor" model and Christaller's (1933) central place theory. Subsequent papers have explored firms that compete on price and/or location (e.g., Capozza and Van Order, 1978; Anderson, de Palma, and Thisse, 1992; Beckmann, 1999).[2] In their models, retail stores mutually compete for their market areas and scale economies. Retail stores must make two crucial decisions: where to locate and the prices of their goods/services. The spatial price competition framework has been adopted by many studies. As Anderson, de Palma, and Thisse (1992) claim, the result of spatial price competition has more far-reaching implications under classical price theory with nonspatial parameters because most stores operate in a spatial environment.

With the volume of previous works related to spatial price competition, how can this paper offer new insight? Actually, most previous studies of spatial price competition assume homogeneity of space.[3] Although such an assumption greatly simplifies analyses, in reality, transportation costs depend on the conformations of transport networks (e.g., roads). In addition, cities are generally more complex as they have a center and peripheral areas that are typically strongly tied to it.[4] Therefore, to express such urban structure with transport networks, one must depart from any assumption of homogeneous space.

Assuming an exogenous Manhattan roadway grid with a store at each intersection, Braid (1993) explores two dimensional price competition among stores. Similarly, Braid (2013) analyzes a city with a central intersection and radial roadways extending from the center to the suburb. His work sheds light on competition among stores in a city center and suburban stores using an exogenously determined road network. Following his modeling, we develop a city with spatial price competition, but with stores located on a different exogenous road network that is a combination of ring and radial roads. For example, Braid (2013) assumes only one store at the central intersection, whereas in our setting retail stores in the center compete with neighboring stores on the central ring roads. In addition to the different network, retail stores in central areas

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several countries.) Recently, similar hollowing-out of central business districts has been observed in numerous small towns in Japan, since regulations of commercial development on suburban areas were loosened in the 1990s.

[2] See Fujita and Thisse (2002) for a survey of the related literature.

[3] Several previous studies assumed heterogeneous space. For example, Lai (2001) assumed one-dimensional space in which transportation costs vary with the trip direction. Fujita and Thisse (1986) assumed endogenous location choice of consumers' residences. But they do not suppose that such an urban structure has a city center and outskirts; therefore, they cannot analyze the dichotomy of store location that we examine here.

[4] To explain endogenously how the city center and outskirts arise, some studies account for communication externalities (Beckmann, 1976), interaction among stores (Borukov and Hochman, 1977; O'Hara, 1977), and interdependence of stores and workers (Ogawa and Fujita, 1980; Imai, 1982; Fujita and Ogawa, 1982). The current paper sets the center and the outskirts exogenously.

and outskirts compete by selling imperfect substitutes in our model, and this substitutability is important.

Many papers (e.g., Capozza and van Order, 1977; Mulligan and Fik, 1994) introduce spatial price competition by constructing an unbounded one-dimensional market (a circular market). Based on those models, they investigate the extent to which spatial price competition theory replicates the results achieved using classical non-spatial theory. Results show that some outcomes of spatial comparative statics differ from those in the non-spatial model. Furthermore, the results are affected by the assumptions of stores' conjectures in relation to their rivals' pricing and location choices.

The assumption of stores' conjectures affects the nature and existence of price-location equilibria. Three typical assumptions of stores' conjectures which were often adopted in past papers are the following: Löschian competition (1954), whereby each store assumes its market area to be fixed and sets prices as a monopolist within its market area; Hotelling-Smithies competition, following Hotelling (1929) and Smithies (1941), whereby each store assumes the prices of competitors are fixed; and Greenhut-Ohta competition (1973), whereby each store assumes that the price at the edge of the market area (frontier price) is fixed.[5]

In our model, spatial price competition occurs among stores located along ring roads and radial roads of a city. The assumption of stores' conjectures in our model follows Hoteling-Smithies competition because the most common market structure resembles it. Indeed, in exploring typical price conjectures, Cappoza and van Order (1978) argue that Hotelling-Smithies competition is the most natural. Our method of the derivation of equilibrium is based on that of Capozza and van Order (1977), which is designated hereinafter as CVO.[6]

The current paper's model extends the CVO model in the following two ways. The first extension is the assumption of space. CVO assumes homogeneous, one-dimensional space with constant unit transportation costs in all directions. We introduce a simple road network that includes both radial and ring roads in a city. The second extension is the number of commodities. While CVO assumes only one homogeneous commodity in the market, our model assumes that the goods supplied by stores in the city center have a certain degree of substitutability with the goods supplied by stores in the outskirts, as stores in the city center offer goods or services of different types from those supplied by stores in the outskirts.

These simple extensions from CVO enable us to analyze the location of retail stores scattered throughout both the city center and the suburbs. Using the constructed model, we conduct an analysis of how transportation changes the locations of stores. One of our results show that transportation improvements in the outskirts do not always cause a decline of central city commerce and/or suburbanization of commerce under spatial price competition among stores.

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[5] Pricing under spatial competition in each of these three alternative types is explained by Greenhut, Hwang and Ohta (1975), based on fixed firm location, and by Capozza and Van Order (1977), based on free location of firms. Ohta (1980, 1981) modifies the presentation by Capozza and Van Order (1977) by altering the assumption of the demand function. Ohta and Wako (1988) and Hwang, Mai, and Ohta (1993) analyze pricing, welfare, and firms location in these spatial competition models. Mulligan and Fik (1994) derive medium-run and long-run equilibria under these three price conjectures.

[6] Our main intention is to clarify the relationship between retail store location and transportation improvements; therefore, CVO's exogenous conjectures assumption is appropriate for mathematical tractability.

The remainder of this paper is organized as follows. In Section 2, we construct a location model of retail stores. In Section 3, we derive the location equilibrium of stores. Section 4 shows the condition of dynamic stability of the location equilibrium, which is necessary for investigation of the change in the equilibrium with transportation improvements. Section 5 presents our analyses of the change in market area that occurs concomitantly with transportation improvements and the prosperity or decline of the central city and the outskirts. Section 6 concludes.

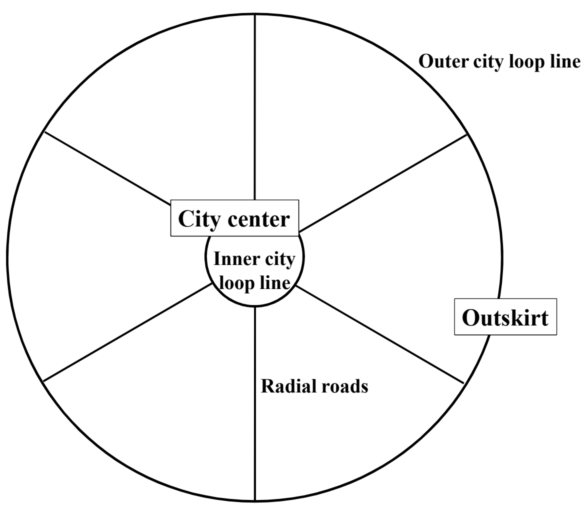
2. THE MODEL

Most real cities have one city center and outskirts surrounding the city center (e.g., London). In the city center, roads are dense and radial roads connect the city center and outlying areas. The outskirts have one or a few outer-city ring roads. Consumers live in both the city center and the outskirts, but the majority live in the suburbs.

Based on such a typical urban structure, we assume a road network for the model as depicted in Figure 1, which is a simplified description of reality for our theoretical analysis. The model has two ring roads: the inner ring road and the outer ring road. They are connected by some radial roads. Stores locate along both ring roads, whereas consumers reside along the outer ring with constant density.[7] The inner ring road can be regarded as the "city center," whereas the outer ring road can be regarded as the "outskirts."

We assume goods of two kinds that have a certain level of mutual substitutability: good *C,* which is supplied by stores on the inner ring road, and good *O* which is supplied by stores on the outer ring road. This assumption intends to reflect real life, by which groceries, daily necessities, and everyday clothing are supplied at suburban stores near consumers' houses, and luxury

Figure 1: City Shape



Note: Residents live only along the outer ring road.

[7] This assumption is for simplicity. In real cities, consumers reside in the center as well as the outskirts. The ratio of residents who live in the center relative to the suburbs is typically quite small for cities above a certain size. In Japan, this assumption is acceptable for all cities with population above 50,000. Our target is such a city.

commodities including high-quality food items, high-quality clothes, and jewelry are supplied in the city center.[8] Usually, good *C* and good *O* have a certain level of substitutability. Details of the model are described in more detail below.

2.1 Consumers

We follow the setting of CVO and assume linear demand function of good *C* and good *O* with respect to their own generalized costs and the generalized cost of the substitute good. The generalized cost is the sum of the price and the transport cost for shopping.

The generalized costs for good *C* and good *O* are calculated as follows. When suburban consumers purchase good *C*, they travel through the outer ring road and pass through the nearest radial road in order to take the inner ring road to the nearest store. Therefore, the generalized cost of good *C* is represented as $p^c + t^o u^r + t^r + t^c u^c$, where p^c signifies the price of good *C*, t^j ($j \in \{c, o\}$) is the unit transportation cost of ring road *j* (center ring or outer ring), u^r represents the distance on the outer ring from the consumer's residence to the entrance of the nearest radial road, t^r denotes the transport cost on the radial road between the entrance and the exit, and u^c expresses the distance on the inner ring from the exit of the radial road to the nearest store on the inner ring. On the other hand, when consumers who reside in the outskirts purchase good *O*, they merely drive along the outer ring road to the nearest store. The generalized cost of good *O* is represented as $p^o + t^o u^o$, where p^o signifies the price of good *O* and where u^o represents the distance on the outer ring between the resident and the nearest store.

The amount of the unit transportation cost, t^j ($j \in \{c, o\}$), of ring road *j* depends on the transportation mode. The transport mode on the ring and radial road can be of any kind, but trips on the outer ring road and the radial road are typically made using cars or public transportation, whereas trips on the inner ring road are made on foot or via public transit.

To set simple demand functions, we introduce two simplifications without loss of generality. The first uses average generalized costs for all residents as the costs for the substitute good in the demand functions. Strictly speaking, the transport cost for the purchase of the substitute goods differs among consumers because consumers reside at different points on the outer ring. But accounting for such different costs is too complex and matters little when aggregate demand is considered. Therefore, the costs are simplified as the average transportation cost for all consumers.[9]

Second, transport costs on the outer ring road between each consumer and the nearest entrance to the radial road is considered to be zero (i.e., it is eliminated from the calculus). This simplification is justified if we simplify the model by assuming that there are infinitely many radial roads instead of the six shown in Figure 1. Thus, a consumer's trip to buy good C does not

[8] This difference between good *C* and good *O* might come from the difference in land rents, transportation costs, and other things. In any case, we treat this difference between the two goods as exogenous.

[9] This simplification would not be valid if the paper explored individual demand. For present purposes, only the aggregated demand of all individuals is required. Therefore, this simplification does not generate critical biases.

involve any travel distance on the outer ring road.[10] The generalized cost of good C is, thus, $p^c + t^r + t^c u^c$.

The two assumptions described above imply that the demand function of each good is a linear function with respect to the generalized cost of the good and that of the substitute good. The demand function of good *O* for residents living on the outer ring with the distance u^o from the nearest store is

$$(1) \qquad x^o = a^o + \underbrace{b^{oc}\left(p^c + t^r + \frac{1}{2}t^c R^c\right)}_{\text{substitution effect}} - \underbrace{b^{oo}\left(p^o + t^o u^o\right)}_{\text{own price effect}},$$

and the demand function of good *C* for a resident on the outer ring is

$$(2) \qquad x^c = a^c + \underbrace{b^{co}\left(p^o + \frac{1}{2}t^o R^o\right)}_{\text{substitution effect}} - \underbrace{b^{cc}\left(p^c + t^r + t^c u^c\right)}_{\text{own price effect}},$$

where R^j ($j \in \{c, o\}$) is the interval between a store and the market boundary of the market area of the store (i.e. half of the so-called "market area") and a^i, b^{ij} ($i, j \in \{c, o\}$) are parameters (a^i, $b^{ij} > 0$). The expressions in parentheses in the second term of Equations (1) and (2) states the generalized cost of the substitute good, whereas the expressions in parentheses in the third term expresses that of the good itself. It is noteworthy that the good's elasticity of its own price is normally greater than the absolute value of elasticity of the substitute good's price. If they are equal, then they are perfect substitutes. Therefore, we assume $b^{cc} \geq b^{co}$ and $b^{oo} \geq b^{oc}$.

In Equations (1) and (2), defining

$$\alpha^o \equiv a^o + b^{oc}\left(p^c + t^r + \frac{1}{2}t^c R^c\right) \text{ and } \alpha^c \equiv a^c - b^{cc}t^r + b^{co}\left(p^o + \frac{1}{2}t^o R^o\right)$$

enables them to be represented in the identical functional form[11] as

$$(3) \qquad x^j = \alpha^j - b^{jj}\left(p^j + t^j u^j\right) \quad (j \in \{c, o\}).$$

Equation (3) is equivalent to the CVO model. Therefore, we can analyze the model which has two substitute goods, using fundamentally the same method as that used for CVO, which has only one good.[12] Equation (3) implies that the substitution effect, which is neglected in CVO,

[10] In our model, the access to the radial road is only one arterial outer ring road. However, in reality, each consumer has other access roads to a radial road, which is from bypass to back street. Therefore, even if the density of radial roads is not so high, they can reach the nearest radial road with only a slight cost.

[11] Because α^j is not a function of u^j, Equation (3) has an identical functional form for the demand functions of good C and good O under the same transportation costs.

[12] Without this property, our model would be quite intractable. The two simplifications of demand functions enable this property.

shifts the demand function of CVO just upwards or downwards through the change in the intercept α^j ($j \in \{c,o\}$).

2.2 Stores

Stores in the city center and the outskirts have scale economies. The cost function of stores with scale economies is assumed as

$$C^j = c^j X^j + f^j \quad (j \in \{c,o\}),$$

where c^j is the marginal cost, f^j is the fixed cost, and X^j is the total demand for goods supplied by a store. Because c^j is fixed with the fixed cost f^j, each store has scale economy.

Setting the consumer density in the outer ring equal to 1 for simplification, the total demand of store X^j can be derived by summing up the demand of each consumer in the store's market area as

$$X^j = 2\int_0^{R^j} x^j du^j = 2R^j \bar{x}^j ,$$

where $\bar{x}^j = \alpha^j - b^{jj}\left(p^j + t^j R^j/2\right)$ represents the average demand in the market area.[13]

The profit of each store is derived as

$$\Pi^j = p^j X^j - C^j = 2R^j\left(p^j - c^j\right)\bar{x}^j - f^j . \quad (4)$$

2.3 Store pricing and market area

Each store maximizes profit under the conditions of free entry and free relocation. We assume the following:

Assumption 1 (Cost-minimizing Shoppers): Consumers travel to the store with the lowest generalized cost when they go shopping.

Assumption 2 (Zero Conjectural Variations): Each store chooses a price and location given the price and the location of the competitors. This assumption corresponds to Hotelling-Smithies competition,[14] which is called "zero conjectural variation,"

Assumption 3 (Perfectly Competitive Market): Store locations do not change in the short run.[15] In the long-run, however, stores continue to enter the market until profits of all

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[13] Substituting $u^j = R^j/2$ into (3) yields $\bar{x}^j$.

[14] Each store believes its neighboring competitors will react to its price changes. Capozza and Van Order (1989) indicates that this reaction, or price conjecture, is endogenous and derives from profit-maximizing behavior. Mulligan and Fik (1995) illustrate endogenous price conjectures in one-dimensional markets. However, our model has "two one-dimensional markets" linked by radial roads. Introducing endogenous price conjectures makes the model quite complicated. Therefore, we introduce Hotteling–Smithies competition in our model.

[15] Mulligan and Fik (1994, 1995) outlined a method for considering location conjectures. They assume three location conjectures: neighboring stores move closer to the new entrant, remain at their present location, or move further from the new entrant. The Hottelling-Smithies model assumes that neighboring stores remain at their present location.

stores are driven to zero because new entrants expect that the market area of each store will be equalized through relocation of all stores.[16]

If a store raises its price, then its market area shrinks due to price competition with neighboring stores. We derive the relationship between the change in price and the change in the market area. For a consumer who lives at the frontier of the i^{th} store's market area and the $i+1^{\text{th}}$ store's market area, the sum of the price and transportation cost of store i and store $i+1$ is $p_i^j + t^j R_i^j = p_{i+1}^j + t^j R_{i+1}^j$. Differentiating with respect to p_i^j and applying Assumption 2 (i.e., $dp_{i+1}^j / dp_i^j = 0$) and Assumption 3 (i.e., $dR_i^j / dp_i^j = -dR_{i+1}^j / dp_i^j$) yields the following equation:

$$\frac{dR_i^j}{dp_i^j} = -\frac{1}{2t^j}, \tag{5}$$

as in CVO. Equation (5) represents the relationship between the change in price and the change in the market area. It is noteworthy that all stores are assumed to be identical in each ring for simplification. Based upon this assumption, Equation (5) holds for any firm i; subscript i is omitted hereinafter because firms are symmetric on each ring.

3. LOCATION EQUILIBRIUM OF FIRMS

Location equilibrium in the model is achieved when each store maximizes profits under conditions of free entry and pricing. In this section, we derive the price and the market area of stores in equilibrium. The profit maximization condition and free entry conditions of all stores are presented as follows.

The profit-maximization condition of each store is

$$\frac{d\Pi^j}{dp^j} = 0 \ \ (j \in \{c, o\}).^{17} \tag{6}$$

We derive two loci to describe the relationship between price and market area of stores: the "profit-maximization locus (*PML*)" and the "zero-profit locus (*ZPL*)". Substituting Equation (4) into Equation (6) gives the relationship between the change in the price (dp^j) and the change in the market area (dR^j) under the profit-maximization condition as

$$\left.\frac{dp^j}{dR^j}\right|_{\frac{d\Pi^j}{dp^j}=0} \begin{cases} > 0 & for \quad R^j < \hat{R}^j \\ < 0 & for \quad R^j > \hat{R}^j \end{cases} \quad \text{and} \quad \left.\frac{d^2 p^j}{dR^{j2}}\right|_{\frac{d\Pi^j}{dp^j}=0} < 0, \tag{7}$$

[16] Eaton (1976) and Eaton and Lipsey (1978) point out that zero profits are not a condition of equilibrium. Indeed, a new entrant might suffer from less profits than existing stores or negative profit under location adjustment process. It is time-consuming in reality, because capital transfers take money and time so the new entrants' conjecture is such that neighboring stores do not change their locations to prevent new entrants. On this conjecture, no store enters the market if it is facing a demand curve that lies below the demand curve of any existing firm, even if existing firms are earning positive profits. Therefore, zero profits are not a condition of equilibrium. But we assume the zero-profit condition for simplification of mathematical tractability for delivering equilibrium. This simplification can be justified as long as retail stores can relocate quickly at a low cost.

[17] The second-order condition $d^2\Pi^j / dp^{j2} < 0$ is necessary so that (6) is the profit-maximization condition. Appendix A shows $d^2\Pi^j / dp^{j2} < 0$ so that (6) is exactly the profit-maximization condition.

where $\hat{R}^j = \alpha^j / b^{jj} t^j - \left(3p^j - c^j\right) / 2t^j$ is the interval of market area for which $dp^j / dR^j = 0$. The derivation of Equation (7) is shown in Appendix A. We designate the locus of $\left(p^j, R^j\right)$ which satisfies Equation (6) as the *PML* Equation (7) shows the shape of the *PML*.

Free entry of stores drives firm profits to zero, which is shown as

(8) $$\Pi^j = 0 \; (\, j \in \{c, o\} \,).$$

Substituting Equation (4) into Equation (8) yields the relationship between the change in price (dp^j) and the change in market area (dR^j) under the zero-profit condition as[18]

(9) $$\left.\frac{dp^j}{dR^j}\right|_{\Pi^j = 0} < 0 .$$

We designate the locus of $\left(p^j, R^j\right)$, which satisfies Equation (8) as the "zero-profit locus (*ZPL*)." Equation (9) shows the shape of the *ZPL*.

Both Equations (6) and (8) should be satisfied simultaneously under the location equilibrium of stores; in fact, *PML* is expected to intersect with *ZPL* for the existence of the equilibrium and the intersections of the two curves are the equilibrium solutions of the model. From Equations (7) and (9), the shapes of the *PML* and the *ZPL* are portrayed in Figure 2. Points A and B in Figure 2 are candidates for equilibrium solutions. Stores set a price, p^{j*}, that maximizes their profits given the price and the location of its competitors. Next, if profits are positive, then the entry of new stores reduces each store's market area. If profits are negative, then the withdrawal of stores expands each remaining store's market area. Accordingly, the adjustment process of the equilibrium point can be represented by the arrows in Figure 2. In conclusion, point B is an unstable equilibrium; only point A is a stable equilibrium, as discussed by CVO.

For simplification of the derivation of the equilibrium, we assume the following.

Assumption 4**:** $R^j < \hat{R}^j$

If Assumption 4 does not hold, that is $R^j \geq \hat{R}^j$, then the circumstances yield the following unrealistic situation. For $R^j \geq \hat{R}^j$, $2x_R^j - b^{jj}\left(p^j - c^j\right) < 0$, where x_R^j denotes

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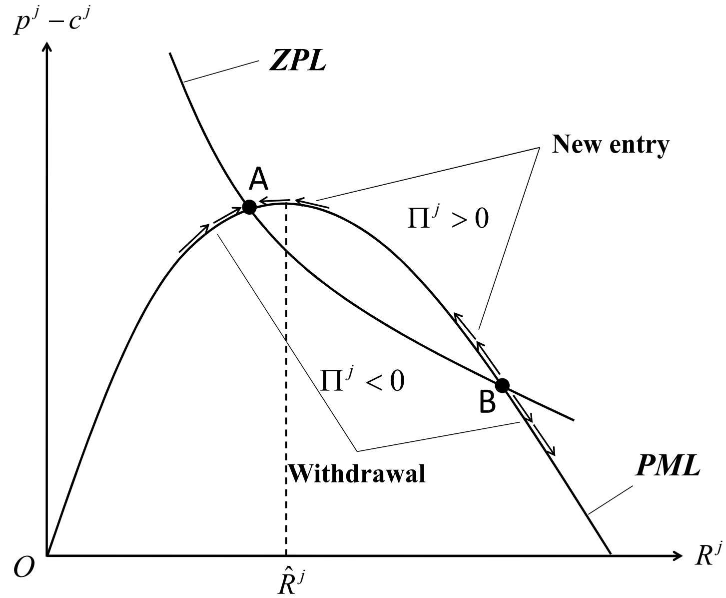
[18] Substituting Equation (4) into Equation (8) yields $2R^j\left(p^j - c^j\right)\bar{x}^j - f^j = 0$. Total differentiating with respect to p^j and R^j results in $2R^j\left[\bar{x}^j - b^{jj}\left(p^j - c^j\right)\right]dp^j + 2\left(p^j - c^j\right)x_R^j dR^j = 0$. The first term is positive from Equation (A.1) and the second term in Equation (B.2) is also positive because the demand at the market frontier x_R^j is positive and, therefore, $\left. dp^j / dR^j \right|_{\Pi^j = 0} < 0$.

FIGURE 2: Profit-maximization Locus (*PML*) and Zero-profit Locus (*ZPL*).



the demand at the edge of each market area (see Appendix A). A transformation of this condition yields

$$\frac{\bar{x}^j}{x_R^j} \geq 2 + \frac{p^j - c^j}{2R^j t^j}$$

which shows that the average demand $\bar{x}^j$ is more than twice the demand at the market boundary x_R^j. For that reason, the price elasticity of demand is much larger and each store's market area is sufficiently large. Such a situation seems unrealistic and does not hold for most goods.[19] Therefore, Assumption 4 is appropriate in practice. Assumption 4, $R^j < \hat{R}^j$, implies that $2x_R^j - b^{jj}\left(p^j - c^j\right) > 0$.[20]

4. FIRM'S PROFIT FUNCTION AND THE CONDITION OF DYNAMIC STABILITY

The change in the location of stores with improvements in transportation, which is represented as dR^j/dt^k where $k \in \{r,o\}$, can be derived from the profit-maximization condition (6) and the zero-profit condition (8). This section explains the condition of dynamic stability for price and market area with the change in equilibrium, which is necessary for the derivation of the solution of dR^j/dt^k.

Figure 3 presents the shift of *PML* and *ZPL* after a change in transportation costs. Under Assumption 3, price adjustments must occur more rapidly than location adjustment. That is, firms take measures in the following two stages:

[19] For example, many necessities (e.g., groceries) have relatively inelastic demand.

[20] This condition is used in Appendix C.

Stage 1: Location adjustment

Stores enter the market and decide where to locate, and

Stage 2: Price adjustment

Stores choose a price according to the location decisions of all stores.

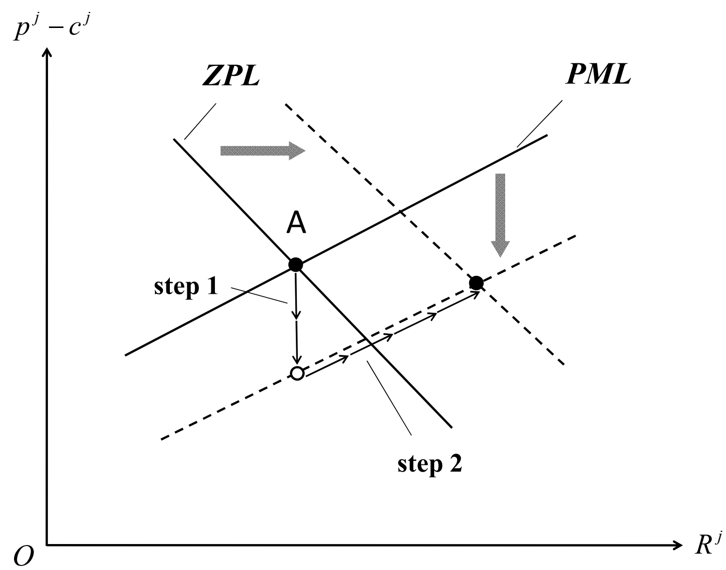
We now solve each of the two adjustment stages in reverse. This adjustment process is expressed by two curves, *PML* and *ZPL*, after the change in transportation costs.

The solution of simultaneous Equations (6) and (8) is obtained using the following two steps. In Stage 2, p^{j*} is derived by Equation (6) with given R^j.[21] In Stage 1, by substituting derived $p^{j*}\left(R^j\right)$ in Stage 1 into Equation (8), R^j is derived. We assume a Nash equilibrium for the stability condition.

As explained in the adjustment process presented above, firms' profits are a function of p^{j*} and R^j. Moreover, p^{j*} is a function of R^j. We define the profit function of firms in each region as

$$\Pi^c\left(R^c, R^o, p^{c*}\left(R^c, R^o\right), p^{o*}\left(R^c, R^o\right)\right) \text{ and } \Pi^o\left(R^c, R^o, p^{c*}\left(R^c, R^o\right), p^{o*}\left(R^c, R^o\right)\right).$$

FIGURE 3: Adjustment Process of Equilibrium.



[21] R^j is half of the market size of each store. Because stores are homogenous in each ring of the current model, the market size is symmetric for all stores in each ring. Therefore, R^j also expresses the distance between the two stores. "Given R" in Stage 1 implies fixing the location of stores.

4.1 Condition of dynamic stability for price (Stage 2: price adjustment)

The dynamic equation of the price adjustment process under the exogenous change in transportation cost, t^k ($k \in \{r,o\}$), is represented as the following determinant

$$\begin{pmatrix} \dot{p}^{c*} \\ \dot{p}^{o*} \end{pmatrix} = \beta \begin{pmatrix} \frac{d\Pi^c}{dp^c}\left(\bar{R}^c, \bar{R}^o, p^c, p^o\right) \\ \frac{d\Pi^o}{dp^o}\left(\bar{R}^c, \bar{R}^o, p^c, p^o\right) \end{pmatrix},$$

where β denotes a positive constant. This equation implies that a store increases (*respectively.* decreases) its price p^j if the increase of its own price increases (*respectively* decreases) its own profits. The condition of dynamic stability for price is

(10) $$\begin{vmatrix} \Pi^c_{p^c}{}' & \Pi^c_{p^o}{}' \\ \Pi^o_{p^c}{}' & \Pi^o_{p^o}{}' \end{vmatrix} > 0 \quad \text{and} \quad \Pi^c_{p^c}{}' + \Pi^o_{p^o}{}' < 0,$$

where $\Pi^{j\prime}_e = \partial/\partial e\left(\partial\Pi^j/\partial p^j\right)$.

4.2 Condition of dynamic stability for market area (Stage 1: location adjustment)

The dynamic equation of the market-area adjustment process under the exogenous change in transportation costs t^k ($k \in \{r,o\}$) is represented as

$$\begin{pmatrix} \dot{R}^c \\ \dot{R}^o \end{pmatrix} = -\gamma \begin{pmatrix} \Pi^c\left(R^c, R^o, p^{c*}\left(R^c, R^o\right), p^{o*}\left(R^c, R^o\right)\right) \\ \Pi^o\left(R^c, R^o, p^{c*}\left(R^c, R^o\right), p^{o*}\left(R^c, R^o\right)\right) \end{pmatrix},$$

where γ is a positive constant. This equation implies that the market area of a store, R^j, is diminished by of the entry of new stores when the store has positive profits (i.e., $\Pi^j > 0$) and that if $\Pi^j < 0$ then R^j is increased dynamically by the withdrawal of stores (See Fig. 2). The condition of dynamic stability for market area is

(11) $$\begin{vmatrix} \Pi^c_{R^c} & \Pi^c_{R^o} \\ \Pi^o_{R^c} & \Pi^o_{R^o} \end{vmatrix} > 0 \quad \text{and} \quad \Pi^c_{R^c} + \Pi^o_{R^o} > 0,$$

where $\Pi^j_e = \partial\Pi^j/\partial p^j$.

5. CHANGE IN THE MARKET AREA WITH TRANSPORTATION IMPROVEMENTS

Transportation improvements change the equilibrium. In this section, we derive the change in the market area with transportation improvements in the radial and ring roads.

5.1 Change in the market area with radial road improvement

We derive the change in the market area with a radial road improvement, dR^c/dt^r and dR^o/dt^r. The effects are derived as the solutions of the simultaneous equations derived through the total differentiation of Equation (8) with substitution of p^{j*} and $dt^o=0$. As a result, we obtain $dR^c/dt^r>0$ and $dR^o/dt^r<0$. These derivations are shown in Appendix B. These derived results are rewritten as the following proposition.

> **Proposition 1 (Effects of radial road improvement):** *Radial road improvements increase the number of stores in the city center while decreasing the number of stores in the outskirts.*

This result is straightforward. Radial road improvements decrease transportation costs to stores in the city center. However, it does not decrease transportation costs to stores in the outskirts. Therefore, radial road improvements make stores in the city center more attractive for consumers. For that reason, Proposition 1 is obtained.

5.2 Change in the market area with improvement of the outer ring road

We derive the change in the market area with an improvement of the outer ring road, dR^c/dt^o and dR^o/dt^o. These are derived as the solution of the simultaneous equations derived through the total differentiation of Equation (8) with substitution of p^{j*} and $dt^r=0$. The derivation is shown in Appendix C.

As explained in Appendix C, whether dR^c/dt^o and dR^o/dt^o are positive or negative depends on the sign and the size of $\Pi^o_{t^o}$, partial differentiation of the profit function. In the case of $\Pi^o_{t^o}<0$, the sign of dR^c/dt^o and dR^o/dt^o is determined as $dR^c/dt^o<0$ and $dR^o/dt^o>0$. The derived result is written in Proposition 2.

> **Proposition 2 (Effects of outer ring improvement ($\Pi^o_{t^o}<0$)):** *If $\Pi^o_{t^o}<0$, then improvement of the outer ring decreases the number of stores in the city center while increasing the number of stores in the outskirts.*

Proposition 2's result is intuitive. Improvement of transportation in the outskirts decreases transportation costs to stores located in the outskirts. Consequently, improvement of transportation in the outskirts makes stores in the outskirts more attractive to consumers. This consequence bears out the existing claim that transportation improvements in outlying areas are a main cause of the hollowing-out of commercial centers.

But in the case of $\Pi^{o}_{t^o} > 0$, in contrast to the condition of Proposition 2, the sign of dR^c/dt^o and dR^o/dt^o cannot be determined. It is affected by the relative magnitudes of $\Pi^{c}_{R^o}$, $\Pi^{c}_{R^c}$, and $\Pi^{o}_{t^o}$. If profits are sufficiently large, then $dR^c/dt^o > 0$ and $dR^o/dt^o < 0$. If they are sufficiently small, then $dR^c/dt^o < 0$ and $dR^o/dt^o > 0$. This is summarized as Proposition 3 shown below.

Proposition 3 (Effects of outer ring improvement ($\Pi^{o}_{t^o} > 0$)): *If $\Pi^{o}_{t^o} > 0$ and if $\Pi^{c}_{R^o}$, $\Pi^{c}_{R^c}$, and $\Pi^{o}_{t^o}$ are sufficiently large, then an improvement of the outer ring increases the number of stores in the city center while decreasing the number of stores in the outskirts. If $\Pi^{o}_{t^o} > 0$ and $\Pi^{c}_{R^o}$, $\Pi^{c}_{R^c}$, and $\Pi^{o}_{t^o}$ are sufficiently small, then an improvement of the outer ring decreases the number of stores in the city center while increasing the number of stores in the outskirts.*

As stated in Proposition 3, the sign of dR^c/dt^o and dR^o/dt^o is the same as that in Proposition 2 if $\Pi^{o}_{t^o} > 0$ and if $\Pi^{c}_{R^o}$, $\Pi^{c}_{R^c}$, and $\Pi^{o}_{t^o}$ are sufficiently small. Accordingly, if $\Pi^{o}_{t^o}$ is negative or sufficiently small, then an improvement of the outer ring road implies that the number of stores in the city center decreases while the number of stores in the outskirts increases.

But when $\Pi^{o}_{t^o} > 0$, if $\Pi^{c}_{R^o}$, $\Pi^{c}_{R^c}$, and $\Pi^{o}_{t^o}$ are sufficiently large, then a counterintuitive result arises. The improvement of the outer ring road increases the number of stores in the city center while it decreases the number of stores in the outskirts. That is to say, transportation improvements of outlying areas promote the agglomeration of commerce into the city center. This paradoxical consequence is in contrast[22] to the existing claim that transportation improvements in outlying areas are a main cause of the hollowing out of commercial centers.

This result comes from the following three steps: an improvement in the outer ring directly improves the comparative advantage of the stores locating in the outskirts to the stores locating in the city center (Step 1). This effect increases (*respectively* decreases) the number of stores in the outskirts *(respectively* city center). At the same time, the improvement in the outer ring road accelerates competition among the stores in the outskirts, and reduces the profits of stores in the outskirts (Step 2). This effect decreases the number of stores in the outskirts, and this reduction in the number of stores in the outskirts increases the generalized cost of good *O*, depending on the demand elasticity of good *O*, and thus worsens the comparative advantage of stores locating in the outskirts (Step 3). As a result, an improvement in the outer ring could increase the number of stores in the city center.

We investigate a sufficient condition under which the paradoxical consequence arises. This is summarized as follows.

[22] Such seemingly perverse consequences are often derived via models with distorted prices (e.g., monopolistic competition models).

Sufficient condition for the occurrence of Proposition 3: $\Pi^{o}_{t^o} > 0$, and $\Pi^{c}_{R^o}$, $\Pi^{c}_{R^c}$, and $\Pi^{o}_{t^o}$ are sufficiently large if b^{cc} and b^{oc} are sufficiently small, whereas b^{co} and b^{oc} are sufficiently large.[23]

Because it is assumed that $b^{cc} \geq b^{co}$ and $b^{oo} \geq b^{oc}$, these conditions are transformed such that "$b^{cc} - b^{co}$ and $b^{oo} - b^{oc}$ are sufficiently small." In other words, we can say that the necessary condition for the paradoxical consequence is that the substitutability between good *C* and good *O* is sufficiently large. To understand this result intuitively, we show what occurs under strong substitutability using a numerical simulation.

Table 1 shows the change in the sign of dR^c/dt^o and dR^o/dt^o in two different numerical simulations.[24] One is Case 1 (small substitutability case): that improvement of the outer ring road decreases the number of stores in the city center while increasing the number of stores in the outskirts. This corresponds to Proposition 2. The other is Case 2 (large substitutability case): that improvement of the outer ring increases the number of stores in the city center while decreasing the number of stores in the outskirts. This corresponds to the paradoxical consequence in Proposition 3. The result of the numerical simulation, as shown in Table 1, demonstrates the following three things.

Table 1: Numerical Simulation

	Case 1 (small substitutability)	Case 2 (large substitutability)
$b^{cc} - b^{co}$	6	5
dp^c/dt^o	7.79×10^{-4}	1.17×10^{-3}
dR^c/dt^o	-2.32×10^{-2}	1.02×10^{-2}
$d\left(p^c + t^r + t^c R^c/2\right)/dt^o$	-5.03×10^{-3}	3.71×10^{-3}
dp^o/dt^o	-1.07×10^{-3}	−0.841
dR^o/dt^o	0.681	−7.78
$d\left(p^o + t^o R^o/2\right)/dt^o$	0.807	−1.65

(Parameter setup: $a^c = 100, a^o = 120, b^{cc} = 8, b^{oo} = 7, b^{oc} = 2, c^c = 8, c^o = 12,$ $f^c = 30, f^o = 15, t^c = 0.5, t^o = 0.7, t^r = 1$)

[23] The proof is straightforward. A decrease in b^{cc} and an increase in b^{co} makes $\Pi^{c}_{R^o}$ and $\Pi^{c}_{R^c}$ large (see Equations (B.1.1) and (B.1.2) in Appendix B). A decrease in b^{oo} and an increase in b^{oc} makes $\Pi^{o}_{y^o}$ large (see Equation (C.1.2) in Appendix C).

[24] The solutions of other variables are shown as the supplement on the authors' websites, see e.g., <http://ai.u-shizuoka-ken.ac.jp/user/kishi/>. The result of numerical simulation implements Assumption 4.

Main results of the numerical simulation

1-1: $dp^c / dt^o > 0$ and $dp^o / dt^o < 0$ in Case 1 and Case 2

1-2: $dR^c / dt^o < 0$ in Case 1 whereas $dR^c / dt^o > 0$ in Case 2

1-3: $d\left(p^c + t^r + t^c R^c / 2\right) / dt^o < 0$ and $d\left(p^o + t^o R^o / 2\right) / dt^o > 0$ in Case 1 whereas $d\left(p^c + t^r + t^c R^c / 2\right) / dt^o > 0$ and $d\left(p^o + t^o R^o / 2\right) / dt^o < 0$ in Case 2.

Knowing what occurs in Case 1 and Case 2, it is helpful to examine **main result 1-1**, **1-2**, and **1-3** simultaneously. In Case 1, an improvement of the outer ring road decreases the price of good *C*. Retail stores in the city center become more competitive. However, it concurrently decreases the number of stores in the city center. The result shows that a retail store in the city center becomes more monopolistic. Thereby, the generalized cost for good *C* increases because the increase in travel costs resulting from the decrease in the number of stores is greater than the decrease in price, as shown in **main result 1-3**. Focusing on the outskirts, an improvement of the outer ring road increases the price of good *O*. Retail stores in the outskirts become more monopolistic. However, the improvement concurrently increases the number of stores in the outskirts. The results in Table 1 show that retail stores in the outskirts become more competitive. Results indicate that generalized costs for good *O* decrease because the decrease in travel cost caused by the increase in the number of stores is greater than the increase in price, as presented in **main result 1-3**.

Such results, which are demonstrated by Case 1 in Table 1 and **Proposition 2**, are observed in real cities. One empirical instance is shown by Leslie and Ó hUallacháin (2006), in which the CBD in Phoenix has few retail stores and new entry of retail stores is concentrated in some suburban areas with good freeway access. This result means that traffic improvement in a suburban area increases the number of retail store in outskirts, as shown in Case 1. Another example of Case 1 is urban “food deserts,” where inner-city grocers do not provide a wide array of goods due in part to a lack of competition.

In Case 2, an improvement of the outer ring road decreases the price of good *C*. Evidently, retail stores in the city center become more competitive. Additionally, the ring improvement concurrently increases the number of stores in the city center, which differs from Case 1. Therefore, the generalized cost for good *C* decreases, as shown in **main result 1-3**. Focusing on the outskirts, an improvement of the outer ring road increases the price of good *O*. This finding indicates that retail stores in the outskirts become more monopolistic. Additionally, ring improvements concurrently decrease the number of stores in the outskirts, which differs from Case 1. Consequently, the generalized cost for good *O* increases, as shown in **main result 1-3**.

The examination above presents us with a typical circumstance that causes the paradoxical consequence shown in Proposition 3. If retail stores in the outskirts are more spatially monopolistic than those in the city center, then an improvement of the outer ring road promotes their spatially monopolistic situation. Furthermore, substitutability between good *C* and good *O* is sufficiently large; therefore, a large amount of demand for good *O* shifts the demand for good *C*. In the city center, retail stores are more competitive than those in the outskirts.

Therefore, more retail stores will start operating in the city center, although some retail stores withdraw from operating in the city outskirts.

We show theoretically under what condition Case 2 in Table 1 occurs. In particular, large substitutability between goods provided in the center and the suburb is important. Unfortunately, there are no empirical studies which observe Case 2. That is probably because the urbanization/suburbanization of cities is caused by various factors. We analyze the effect of transportation improvements, which is one of the various factors. Changes in population and individual attributes, such as income, are other crucial factors. It is difficult to separate the effect of transportation improvements from the effects of other factors. But, our research suggests there are significant effects of transportation improvements on retail store locations.

6. CONCLUSION

We have investigated the relationship among the decline of central commerce, the suburbanization of commerce, and transportation improvements. Transportation improvements in the outskirts are regarded as a main cause of the hollowing-out of the commercial center. Our model explains this claim theoretically.

We derived two important consequences of transportation improvements. First, radial road improvements contribute to agglomeration of commerce into the city center. Improving a radial road, which connects the city center and the outskirts, improves access to the city center and thereby causes an increase in the demand for good C, which is supplied in the city center, while decreasing the demand for good O, which is supplied in the outskirts. Consequently, the road improvement increases the number of stores in the city center while decreasing the number of stores in the outskirts.

Second, it is proven theoretically that improvements of the outer ring does not always cause the decline of central commerce and the suburbanization of commerce. This is in contradiction to an existing claim. A necessary condition for this contradiction to be true is that the substitutability between good C and good O is sufficiently large. Under this condition, an improvement of the outer ring road decreases the average generalized cost of good C while increasing that of good O. This result is attributable merely to the scale economies of retail stores and their spatial price competition. In recent years, substitutability between commerce in the city center and commerce in the outskirts has been increasing because of mass production. Therefore, transportation improvements in outlying areas might not be a cause of hollowing-out and suburbanization of commerce in the future.

There are several considerations for further analysis. First, we have to consider the detail of shopping activity of consumers. A certain proportion of shopping trips are multipurpose trips, which are combined with commuting trips or shopping trips for other goods. O'Kelly (1983) shows using the data on consumers' travels and the retail sizes in Hamilton, Ontario, that multi-purpose shopping trips relate closely to retail store location. Therefore, it would be reasonable to introduce multipurpose shopping activity into the model.

Second, we should explore how stores' conjectural variations affect our conclusion. Assuming some other exogenous conjectural variations or making conjectural variation endogenous should be considered in the future if mathematical tractability permits.

Third, future work should take into account the discontinuous location changes of firms. We assume continuous changes in a firm's location and neglect discontinuous changes in the location. With decreasing transportation costs, a firm's location can change drastically (not continuously). A future task should take account of such a drastic change case. For such analysis, we can follow Ikeda, Akamatsu, and Kono (2009), which have succeeded in combining "group-theoretic bifurcation theory" and "computational bifurcation theory" in determining the emergence of the self-organization of population distribution on a ring road.

REFERENCES

Anderson, Simon P., Andre P. Palma, and Jacques F. Thisse. (1992) *Discrete Choice Theory of Product Differentiation*. MIT Press: Cambridge, MA.

Beckmann, Martin J. (1976) "Spatial Equilibrium in the Dispersed City," in Yorgos Y. Papageorgiou (ed.), *Mathematical Land Use Theory.* Lexington, MA: Lexington Books, 117−125.

_____. (1999) *Lectures on Location Theory*. Springer-Verlag: Heidelberg, Germany.

Borukhov, Eli and Oded Hochman. (1977) "Optimum and Market Equilibrium in a Model of a City without a Predetermined Center," *Environment and Planning*, A9, 849−856.

Braid, Ralph M. (1993) "Spatial Price Competition with Consumers on a Plane, at Intersections, and along Main Roadways," *Journal of Regional Science*, 33, 187–205.

_____. (2013) "The Locations of Firms on Intersecting Roadways," *Annals of Regional Science*, 50, 791–808.

Capozza, Dennis R. and Robert Van Order. (1977) "Pricing under Spatial Competition and Spatial Monopoly," *Econometrica*, 45, 1329–1338.

_____. (1978) "A Generalized Model of Spatial Competition," *American Economic Review*, 68, 896–908.

_____. (1989) "Spatial Competition with Consistent Conjectures," *Journal of Regional Science*, 29, 1–13.

Christaller, Walter. (1996) *Central Places in Southern Germany*. (C. Baskin trans.) Prentice–Hall: Englewood Cliffs, NJ.

Eaton, B. Curits. (1976) "Free Entry in One-Dimensional Models," *Journal of Regional Science*, 16, 21–33.

Eaton, B. Curits and Richard G. Lipsey. (1978) "Freedom of Entry and the Existence of Pure Profit," *Economic Journal,* 88, 455–469.

Fujita, Masahisa and Hideaki Ogawa. (1982) "Multiple Equilibria and Structural Transition of Non-Monocentric Urban Configurations," *Regional Science and Urban Economics*, 12, 161–196.

Fujita, Masahisa and Jacques F. Thisse. (1986) "Spatial Competition with a Land Market: Hotelling and von Thünen Unified," *Review of Economic Studies*, 53, 819–841.

_____. (2002) *Economics of Agglomeration: Cities, Industrial Location, and Regional Growth Studies*. Cambridge University Press: Cambridge, MA.

Greenhut, Melvin L., Hong Hwang, and Hiroshi Ohta. (1975) "Observations of the Shape and Relevance of the Spatial Demand Function," *Econometrica*, 43, 669–682.

Greenhut, Melvin L. and Hiroshi Ohta. (1973) "Spatial Configurations and Competitive Equilibrium," *Review of World Economics*, 109, 87–104.

Hotelling, Harold. (1929) "Stability in Competition," *Economic Journal*, 39, 41–57.

Hwang, Hong, Chao-C. Mai, and Hiroshi Ohta. (1993) "Löschian Competition versus Spatial Competition: Price and Welfare Comparisons," *Journal of Regional Science*, 33, 13–25.

Ikeda, Kiyohiro, Takashi Akamatsu, and Tatsuhito Kono. (2012) "Spatial Period-doubling Agglomeration of a Core–Periphery Model with a System of Cities," *Journal of Economic Dynamics and Control*, 36, 754–778.

Imai, Haruo. (1982) "CBD Hypothesis and Economics of Agglomeration," *Journal of Economic Theory*, 28, 275–299.

Lai, Fu-C. (2001) "Sequential Locations in Directional Markets," *Regional Science and Urban Economics*, 31, 535–546.

Leslie, Timothy F. and Breandán Ó hUallacháin. (2006) "Polycentric Phoenix," *Economic Geography*, 88, 167–192.

Mulligan, Gordon F. and Timothy J. Fik. (1994) "Price and Location Conjectures in Medium- and Long-run Spatial Competition Models," *Journal of Regional Science*, 34, 179-198.

_____. (1995) "Consistent Price and Location Conjectures in Spatial Competition Models," *Annals of Regional Science*, 29, 91–109.

Ogawa, Haruo and Masahisa Fujita. (1980) "Equilibrium Land Use Patterns in a Non-monocentric City," *Journal of Regional Science*, 20, 455–475.

O'Hara, Donald J. (1977) "Location of Stores within a Square Central Business District," *Journal of Political Economy*, 85, 1189–1207.

Ohta, Hiroshi. (1980) "Spatial Competition, Concentration, and Welfare," *Regional Science and Urban Economics*, 10, 3–16.

_____. (1981) "The Price Effects of Spatial Competition," *Review of Economic Studies*, 48, 317–325.

Ohta, Hiroshi and Tohru Wako. (1988) "The Output Effects of Spatial Price Discrimination Revisited," *Journal of Regional Science*, 28, 83–87.

Reynolds, Jonathan and Christine Cuthbertson. (2003) *Retail Strategy: The View from the Bridge*. Butterworth-Heinemann: Oxford, U.K.

Smithies, Arthur. (1941) "Optimum Location in Spatial Competition," *Journal of Political Economy*, 49, 423–439.

APPENDIX

A. Derivation of Equation (7)

Substituting Equation (4) and (5) into (6) derives

$$\text{(A.1)} \qquad \frac{d\Pi^j}{dp^j} = 2R^j\left[\bar{x}^j - b^{jj}\left(p^j - c^j\right)\right] - \frac{p^j - c^j}{t^j}x_R^j = 0\,,$$

where $x_R^j = \alpha^j - b^{jj}\left(p^j + t^jR^j\right)$ is demand at the edge of the market ($u^j = R^j$). The second order differential of Equation (A.1) is

$$\text{(A.2)} \qquad \frac{d^2\Pi^j}{dp^{j2}} = -\frac{7}{2}R^jb^{jj} - \frac{1}{t^j}\left[\bar{x}^j - \frac{1}{2}b^{jj}\left(p^j - c^j\right)\right].$$

The first term in Equation (A.2) is negative. The parenthesis in the second term of Equation (A.2) can be represented by the deformation of Equation (A.1) as

$$\bar{x}^j - \frac{1}{2}b^{jj}\left(p^j - c^j\right) = \frac{p^j - c^j}{2R^jt^j}x_R^j + \frac{1}{2}b^{jj}\left(p^j - c^j\right) > 0\,.$$

Therefore, the second order condition is satisfied.

Total differentiating Equation (A.1) with respect to p^j and R^j derives

$$\text{(A.3)} \qquad \frac{dp^j}{dR^j} = \frac{2x_R^j - b^{jj}\left(p^j - c^j\right)}{\bar{x}^j - b^{jj}\left(p^j - c^j\right) + \left(7/2\right)R^jb^{jj}t^j}t^j\,.$$

The denominator in Equation (A.3) is positive because Equation (A.1) shows $\bar{x}^j - b^{jj}\left(p^j - c^j\right) = \left(p^j - c^j\right)x_R^j \big/ 2R^jt^j > 0$. Hence, the sign of the numerator in Equation (A.3) determines the sign of $dp^j \big/ dR^j$, which will be explored as follows.

The interval of the market area, $\hat{R}^j$, which makes $dp^j \big/ dR^j = 0$ is derived by $2x_R^j - b^{jj}\left(p^j - c^j\right) = 0$ as $\hat{R}^j = \alpha^j \big/ b^{jj}t^j - \left(3p^j - c^j\right)\big/ 2t^j$. Therefore, $2x_R^j - b^{jj}\left(p^j - c^j\right) > 0$ if $R^j < \hat{R}^j$ and $2x_R^j - b^{jj}\left(p^j - c^j\right) < 0$ if $R^j > \hat{R}^j$. Thus, the following relationship holds.

$$\left.\frac{dp^j}{dR^j}\right|_{\frac{d\Pi^j}{dp^j}=0} \begin{cases} > 0 & for \quad R^j < \hat{R}^j \\ < 0 & for \quad R^j < \hat{R}^j \end{cases}$$

The denominator in Equation (A.3) is an increasing function with respect to R^j, whereas the numerator in Equation (A.3) is a decreasing function with respect to R^j. Accordingly, the second-order derivative, $d^2p^j \big/ dR^{j2}$, is negative.

B. Derivation of dR^c/dt^r and dR^o/dt^r

dR^c/dt^r and dR^o/dt^r are derived as the solution of the simultaneous equations following total differentiation of Equation (8) with substitution of p^{j*} and $dt^o = 0$:

$$d\Pi^c = \Pi^c_{R^c} dR^c + \Pi^c_{R^o} dR^o + \Pi^c_{t^r} dt^r = 0 \text{ and } d\Pi^o = \Pi^o_{R^c} dR^c + \Pi^o_{R^o} dR^o + \Pi^o_{t^r} dt^r = 0 .$$

dR^c/dt^r and dR^o/dt^r are derived as

$$\frac{dR^c}{dt^r} = \frac{\Pi^c_{R^o}\Pi^o_{t^r} - \Pi^o_{R^o}\Pi^c_{t^r}}{J_m} \text{ and } \frac{dR^o}{dt^r} = \frac{\Pi^o_{R^c}\Pi^c_{t^r} - \Pi^c_{R^c}\Pi^o_{t^r}}{J_m},$$

where $J_m \equiv \Pi^c_{R^c}\Pi^o_{R^o} - \Pi^c_{R^o}\Pi^o_{R^c} > 0$, as shown in Equation (11), is the dynamic stability condition for the market area, which is derived in *Stage* 1, the location adjustment. The denominator of J_m is positive and therefore the sign of the numerator determines the sign of dR^c/dt^r and dR^o/dt^r.

The sign of each term in the numerator can be determined following **Property 1**[25] and **Property 2**[26], which are derived below.

Property 1: $\Pi^c_{R^c} > 0$, $\Pi^c_{R^o} > 0$, $\Pi^o_{R^c} > 0$, $\Pi^o_{R^o} > 0$

Property 2: $\Pi^c_{t^r} < 0$, $\Pi^o_{t^r} > 0$

Under the above properties, $dR^c/dt^r > 0$ and $dR^o/dt^r < 0$.

C. Derivation of dR^c/dt^o and dR^o/dt^o

dR^c/dt^o and dR^o/dt^o are derived as the solution of the simultaneous equations of following the total differentiation of Equation (8) after substituting in p^{j*} and $dt^r = 0$:

$$d\Pi^c = \Pi^c_{R^c} dR^c + \Pi^c_{R^o} dR^o + \Pi^c_{t^o} dt^o = 0 \text{ and } d\Pi^o = \Pi^o_{R^c} dR^c + \Pi^o_{R^o} dR^o + \Pi^o_{t^o} dt^o = 0 .$$

dR^c/dt^o and dR^o/dt^o are derived as

$$\frac{dR^c}{dt^o} = \frac{\Pi^c_{R^o}\Pi^o_{t^o} - \Pi^o_{R^o}\Pi^c_{t^o}}{J_m} \text{ and } \frac{dR^o}{dt^o} = \frac{\Pi^o_{R^c}\Pi^c_{t^o} - \Pi^c_{R^c}\Pi^o_{t^o}}{J_m}.$$

[25] Detailed derivation of Property 1 is specified in supplement on author's web site (http://ai.u-shizuoka-ken.ac.jp/user/kishi/).

[26] Detailed derivation of Property 2 is specified in supplement on author's web site (http://ai.u-shizuoka-ken.ac.jp/user/kishi/).

The denominator of J_m is positive and the sign of the numerator determines the sign of dR^c / dt^o and dR^o / dt^o.

The sign of each term in the numerator can be determined given **Property 1** and the following **Property 3.**[27]

Property 3: $\Pi^c_{t^o} > 0$, the sign of $\Pi^o_{t^o}$ is positive or negative

Property 1 and **Property 3** enable us to specify the sign of the numerator. In the case where $\Pi^o_{t^o} < 0$, the sign of dR^c / dt^o and dR^o / dt^o is such that $dR^c / dt^o < 0$ and $dR^o / dt^o > 0$.

[27] Detailed derivation of Property 3 is specified in supplement on author's web site (http://ai.u-shizuoka-ken.ac.jp/user/kishi/).