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Local Labor Markets in a New Economic Geography Model

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Abstract: Space economy is determined by the interaction between markets and the mobility of production factors. Capital and labor mobility affects the functioning of the product and labor markets in the regions of origin and destination. This feeds back into the earnings of production factor owners and changes the incentives to move through the modification of demand levels in both regions. In this paper, we build a model with two regions and two production factors, labor and capital, *à la* Tabuchi-Helpman in order to embody both capital and labor mobility in a unique model. Considering the conditions for agglomeration and dispersion to arise, we show that the features of the labor market are a key parameter along with well-known trade costs. The results show that, depending on labor market conditions, the industry and the population display a smooth bell-shaped curve of spatial development. Dispersion prevails when trade costs are either high or low, while agglomeration occurs in between. Market integration and factor mobility exacerbate regional disparities.

Keywords: New Economic Geography, local labor market, capital and labor mobility, wage curve
JEL Codes: F43, R15, R12

1. INTRODUCTION

Space economy is determined by the interaction between markets and the mobility of production factors (Krugman, 1991). Capital and labor mobility affect both product and labor markets in the regions of origin and destination. This feeds back to the earnings of factor owners and alters incentives so they move through the level of demand in both regions. The main issue is to figure out how and when agglomeration or dispersion of activities may arise as an unintended consequence of a myriad of decisions made by firms and workers pursuing their own interests. Explaining the spatial organization of the economy is the main focus of New Economic Geography.

New Economic Geography is dominated by two models (Thisse, 2010): (a) *the footloose capital model* in which capital is the only production factor and, hence, the capital endowment of each region results from firms' location decisions and (b) *the mobile labor model* in which the only production factor is labor whose migration is driven by households' utilities. Dynamics of the former model are governed by capital's nominal rate of return only, whereas the dynamics of the latter are driven by nominal wages, market prices, and urban costs, which include commuting and housing costs. However, little work has embodied both capital and labor mobility into a unique model.

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Labor mobility and capital mobility are not equivalent for the space economy (Lindsey, 2011). The former involves the migration of workers, who have and carry both their production and consumption capabilities. Further, labor income is spent exclusively where the labor force settles. Contrarily, while capital provides the benefits of added production capability, its returns need not be spent in the region in which they are generated. That is, capitalists need not live where their capital is invested. Understanding the space economy thus requires the separate analysis of the reasons and the obstacles of population migration and industrial relocation.

Capitalists (stockholders) compel firms to locate in the region that offers the highest capital nominal rate of return and workers settle where they maximize their utilities. They do not face the same dispersion and agglomeration forces that rule their location decisions. The spatial distribution of firms arises from the balancing of two opposite forces: the agglomeration force is generated by each firm's desire for market access, which is best provided by the region with the higher income (*Home Market Effect*), whereas the dispersion force finds its origin in each firm's desire to relax spatial competition in product and labor markets, which is effected by moving away from competitors (*Crowding-out Effect*). The location of households is driven by differences in real incomes between the two regions, which depends on the allocation of capital revenues, the local labor market, the number and price of the products available (*Price Index Effect*), and differences in urban costs which rely on the housing supply and the transport network system in each region.

We consider a model with two regions and one sector with two production factors, labor and capital, *à la* Tabuchi-Helpman (Murata and Thisse, 2005). In our model, firms' revenues are split between capital and labor incomes, depending on the balance of power between workers and capitalists. This balance depends on the way workers set their wages. Blanchflower and Oswald (1994) first reported an inverse relationship between the wages paid to individuals and the unemployment rate in local labor markets, and research on different data sets showed that the unemployment elasticity of pay appears to be quite similar across a wide range of countries and time periods, namely about -0.1. This inverse relationship is called the *wage curve* and is seen as an *empirical law of economics* (Blanchflower and Oswald, 2005). A high degree of joblessness in the labor market reduces the ability of workers to claim a large share of the firms' revenues to be divided. Three main possible explanations for the wage curve are usually given: a bargaining wage model, an insider-outsider model, or an efficiency wage model. The *bargaining model* assumes that unemployment frightens workers. So, when wage bargaining reaches an impasse, workers will need to obtain other jobs. Finding jobs is likely to be harder when the local labor market is depressed. They will compromise and will thus obtain lower wages. A variant of this model relies on the explicit assumption of a trade union that worries about both its employed and unemployed members. An increase in unemployment may tilt the union's preferences towards an increased concern with the number of jobs available and a reduced concern for pay. In the *insider-outsider model*, the existence of high adjustment costs (hiring and firing costs) give greater negotiating power to incumbent workers compared to unemployed workers. If the unemployment rate decreases, adjustment costs will grow and employed workers will be able to obtain higher wages. The third wage curve theory is built on the *efficiency wage model* of Shapiro and Stiglitz (1984). Employers, who can imperfectly monitor worker's productivity, will offer a wage that will discourage workers from shirking. Because the expected penalty for shirking is greater when it becomes harder to find a job, firms can offer a lower wage during times of high unemployment. In our model, we assume a wage curve without mention of the microeconomic grounds. The difference between labor incomes and firms revenues are assumed to be captured by capital owners.

Considering the conditions for agglomeration and dispersion to arise, this paper shows that the features of the labor market are key parameters of well-known trade costs. The results show that, depending on the labor market conditions, the industry and population can display a bell-shaped curve of spatial development. Dispersion prevails when trade costs are either high or low, while agglomeration occurs when trade costs are moderated. For agglomeration to occur, the labor market should not be too tight. If not, the degree of agglomeration depends on commuting and housing costs, defined as urban costs.

Numerous enhancements of the core-periphery model (Krugman, 1991) have suggested the existence of a bell-shaped curve of spatial development as trade costs fall. These enhancements encompass imperfect labor mobility due to workers' different attachments to the region where they live (Tabuchi and Thisse, 2002); the existence of nontradable goods such as land, the costs of which increase as agglomeration occurs and cannot be compensated by a better access to the array of tradable goods (Ottaviano et al., 2002; Tabuchi, 1998); or the spatial fragmentation of firms that takes advantage of differences in technologies, factor endowments, and factor prices across space (Fujita and Thisse, 2006). In this paper, the features of the local labor markets generate the bell-shaped curve of spatial development.

The rest of the paper will proceed as follows. The model is introduced in Section 2. The properties of the spatial equilibrium are derived in Section 3 and Section 4 concludes.

2. THE MODEL

2.1 The Spatial Economy

Consider an economy involving two regions (labeled $r = 1,2$ or $j = 1,2$) and one industrial sector producing numerous varieties, $\boldsymbol{n}_r$, of a horizontally differentiated good. Any variety of this good produced under monopolistic competition and increasing returns to scale can be shipped from one region to the other according to iceberg transportation costs *à la* Samuelson: $\tau > 1$ units of the variety must be sent from the origin for one unit to arrive at the destination.

Each region is formed by a city spread along a one-dimensional space x. All firms located in region r are set up at the Central Business District (CBD) situated at the origin $x = 0$. The economy is endowed with a unit of identical and mobile workers, which settle around the CBD and commute to the CBD for work or leisure. Each worker owns one unit of labor. Let λ denote the fraction of workers residing in region 1 so that the mass of workers in region 1 and 2 is given by $L_1 = \lambda$ and $L_2 = 1 - \lambda$, respectively.

2.2 Consumption

In each region, workers bear urban costs due to housing and commuting. A given worker's utility depends on his/her residential location within the city. They make trade-offs between their consumption of differentiated goods, $\boldsymbol{C}_r(x)$, their consumption of housing, $\boldsymbol{h}_r(x)$, and their accessibility to the CBD, $\boldsymbol{a}_r(x)$. Accessibility includes only time costs associated with getting to and from work, visiting relatives and friends, shopping, and other such activities. Following Oort (1969), accessibility observed directly in the workers' utility function. If workers save travel time through higher speeds or shorter commuting distances to visit more destinations, they increase their utility. In region r, workers located at distance x from the CBD maximize the following utility function:

$$(1) \qquad \boldsymbol{U_r}(x) = \boldsymbol{C_r}(x)^{1-\delta^a-\delta^H}\boldsymbol{h_r}(x)^{\delta^H}\boldsymbol{a_r}(x)^{\delta^a}$$

where δ^a and δ^H are the elasticities of utility with respect to accessibility and housing consumed, respectively.

Households prefer variety. Each household's consumption bundle, $\boldsymbol{C_r}(x)$, is thus comprised of a number of different varieties. As in Krugman (1991), we assume that $\boldsymbol{C_r}(x)$ is defined by a constant elasticity of substitution function. Let $\epsilon > 1$ be the substitution elasticity between two varieties and $\boldsymbol{c_{jr}}(x)$ be the variety produced in region j that is consumed by a household located at distance x from the CBD of region r. Thus,

$$\boldsymbol{C_r}(x) = \left(\boldsymbol{\Sigma_{j=1}^2 n_j c_{jr}}(x)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$$

Accessibility is defined here as the number of journeys to the CBD that a worker can take within the time constraint T. Thus, let $\boldsymbol{v_r}(x)$ be the average speed used by a worker to reach the CBD from location x in region r:

$$\boldsymbol{a_r}(x) = \frac{\boldsymbol{v_r}(x)T}{x}$$

Utility maximization under time and budget constraints, T and $\boldsymbol{Y_r}$:

$$\begin{cases} \boldsymbol{Y_r} = \boldsymbol{P_r C_r}(x) + \boldsymbol{R_r}(x)\boldsymbol{h_r}(x) \\ T = \dfrac{\boldsymbol{a_r}(x)x}{\boldsymbol{v_r}(x)} \end{cases}$$

gives:

$$(2) \qquad \begin{cases} \boldsymbol{c_{jr}}(x) = \frac{1-\delta^a-\delta^H}{1-\delta^a}\left(\boldsymbol{\tau_{jr} p_j}\right)^{-\epsilon}\frac{\boldsymbol{Y_r}}{\boldsymbol{P_r^{1-\epsilon}}} \\ \boldsymbol{h_r}(x) = \frac{\delta^H}{1-\delta^a}\frac{\boldsymbol{Y_r}}{\boldsymbol{R_r}(x)} \\ \boldsymbol{a_r}(x) = \dfrac{\boldsymbol{v_r}(x)T}{x} \end{cases}$$

where $\boldsymbol{P_r}$ is the price index of the differentiated good and $\boldsymbol{R_r}(x)$ is housing rent per areal unit. The price index is the same within a region and:

$$(3) \qquad \boldsymbol{P_r^{1-\epsilon}} = \boldsymbol{\Sigma_{j=1}^2 n_j}\left(\boldsymbol{\tau_{jr} p_j}\right)^{1-\epsilon}$$

where $\boldsymbol{\tau_{jr}} = \begin{cases} \tau \ if \ j \neq r \\ 1 \ if \ j \neq r \end{cases}$ and $\boldsymbol{p_r}$ is the production cost of varieties in region r.

As seen above, workers manage consumption to maximize their utility at a given location. But they also choose their location in order to maximize their utility. As accessibility decreases with distance to the CBD, incentives to move closer to the CBD are important. Near the CBD, competition for land is fiercer and housing prices are higher. Workers bid against one another, paying higher rents for proximity to the CBD based on respective accessibility. In those places, workers can afford less housing. The equilibrium is reached within the city when workers have no incentive to move, namely when all locations within the region yield any particular household an equal level of utility. As workers settle further away from the CBD, they suffer poorer accessibility but can enjoy bigger dwellings since rents lower. Alternatively, as they move closer to the CBD, they enjoy better accessibility but suffer higher rent. Thus, in equilibrium, the accessibility and net rent differentials, which would be obtained from any move, offset one another. Each region is characterized by a fixed amount of housing supply at distance x from the CBD, $\boldsymbol{H_r}(x)$. The housing market entails:

$$\boldsymbol{H_r}(x) = \boldsymbol{h_r}(x)\boldsymbol{L_r}(x) \tag{4}$$

Equation (4) sets the number of workers living at distance x from the CBD. As seen above, workers split up the housing supply so that their utility is constant wherever they settle. We assume that the housing supply is constant within each region, as in Helpman (1998) and Tabuchi (1998). We assume that households move a lot faster within a region than between regions 1 and 2.

2.3 Production

The production of differentiated goods involves economies of scale. These economies of scale are assumed to arise through variety. Each firm produces a single variety[1] with identical technologies under monopolistic competition (Dixit and Stiglitz, 1977). The total number of varieties is thus fixed to the number of firms, $\boldsymbol{n_r}$. Let β denote the fraction of firms settled in region 1 so that the number of firms in regions 1 and 2 is given by $\frac{n_1}{n_1+n_2} = \beta$ and $\frac{n_2}{n_1+n_2} = 1 - \beta$, respectively. The production of a variety requires a fixed amount of capital, K, and a variable amount of labor. Any change in one region's capital endowment induces a corresponding change in the number of firms. To produce $\boldsymbol{q_r}$ units of output, $\boldsymbol{l_r q_r}$ units of labor are required. The profit is as follows:

$$\boldsymbol{\pi_r} = \boldsymbol{p_r q_r} - \boldsymbol{w_r l_r q_r} - K\boldsymbol{r_r}$$

where $\boldsymbol{w_r}$, $\boldsymbol{l_r}$ and $\boldsymbol{r_r}$ are the wage, the quantity of labor required per unit of output (the inverse of productivity) and the capital nominal rate of return. The choice of the price, $\boldsymbol{p_r}$, that maximizes profits is determined by mark-up pricing over marginal costs, which is a standard rule in monopolistic competition:

$$\boldsymbol{p_r} = \frac{\epsilon}{\epsilon-1}\boldsymbol{l_r w_r} \tag{5}$$

Unlike in Krugman (1991), entry and exit of firms in the market are not free. Firms' relocations are enabled by capital mobility, but profits are zero in equilibrium because firms pay returns (rent) to capitalists. Thus, rents on capital are defined as the difference between revenues and labor costs:

$$\boldsymbol{r_r} \equiv \frac{1}{K}(\boldsymbol{p_r q_r} - \boldsymbol{w_r l_r q_r})$$

[1] Because of increasing returns of scale, households' preference for variety, and the unlimited number of potential varieties of goods, no firm chooses to produce the same variety supplied by another firm (Krugman, 1991).

The split of the firms' revenues between capital income and wages is determined by a *wage curve*. This curve illustrates the balance of power between capital owners and workers. It summarizes the fact that a worker who is employed in an area of high unemployment earns less than an identical individual who works in a region with low joblessness.[2] Let $\boldsymbol{S_r}$ be the number of active workers in region r:

$$\boldsymbol{w_r} = \left(1 - \frac{S_r}{L_r}\right)^{-\sigma} \tag{6}$$

where $-\sigma$ is the elasticity of wages to unemployment.

2.4 Income and revenue allocations

We consider each region as an independent jurisdiction that owns the land of its region. This assumption is reasonable as long as there exists no "global government." As a result, housing expenditures are shared between every region's residents. Besides, we assume also that each region allocates an equal amount of capital and redistributes capital revenues throughout its population. Accordingly, each worker receives an income equals to:

$$\boldsymbol{Y_r} = \boldsymbol{w_r}\delta + \frac{1}{2\boldsymbol{L_r}}\boldsymbol{\Sigma}_{\boldsymbol{j}=\boldsymbol{1}}^{\boldsymbol{2}}\boldsymbol{r_j}K\boldsymbol{n_j} + \frac{\delta^H}{1-\delta^a}\boldsymbol{Y_r}$$

where $= \begin{cases} 0 \; if \; unemployed \\ 1 \; if \; employed \end{cases}$. When skilled workers move to a new region, they bring with them both their production and consumption capabilities. As a result, their movements simultaneously affect the size of the labor and product markets in both the origin and destination regions. By contrast, the movement of capital to a region brings with it the benefits of added production capability, but the returns from this capital need not be spent in the same region. Returns of capital are, in this model, split between the two regions, not depending on where firms are settled.[3] This is the main difference between capital and labor mobility.

2.5 Market clearing conditions

The short-term equilibrium is characterized by equilibria of the differentiated good market and the labor market. In the differentiated good market, the production of every firm is consumed either in the region of production or in the other region. The differentiated good market clearing condition is:

$$\boldsymbol{q_r} = \boldsymbol{\Sigma}_{\boldsymbol{j}=\boldsymbol{1}}^{\boldsymbol{2}} \int_{\boldsymbol{X}} \boldsymbol{\tau_{rj}}\boldsymbol{c_{rj}}(x)\boldsymbol{L_j}(x)dx \tag{7}$$

As for the labor market, the market-clearing condition in region r is given by the equality between active workers in the region and the labor needed to produce $\boldsymbol{n_r q_r}$ outputs:

$$\boldsymbol{S_r} = \boldsymbol{l_r}\boldsymbol{n_r}\boldsymbol{q_r} \tag{8}$$

The wage curve guarantees that the number of active workers is always smaller than the total number of workers.

[2] The cost of living is assumed to have no direct impact on the wage curve, but an indirect impact through workers' migrations.

[3] Different assumptions could be made. We decided to split the capital revenues between the two regions, even if more firms or more workers are located in a given region, in order to stress the differences between capital and labor. If the capital revenues are split between the two regions depending on where firms are settled, there is no difference between capital and labor in this model.

The dynamics of our model are driven by the migrations of firms (capital mobility) and workers (labor mobility) between the two regions. Their location choices do not follow the same rationales. Households as capital owners tend to invest their capital where capital revenues, $\boldsymbol{r_r}$, are the highest, while households as workers seek the highest utility. The quantitative dynamics in firms' and workers' migrations are respectively calculated as follows:

$$\frac{\boldsymbol{dn_r}}{\boldsymbol{dt}} = \frac{\boldsymbol{r_r - r_j}}{\omega^n}\Big|_{j\neq r}$$

$$\frac{\boldsymbol{dL_r}}{\boldsymbol{dt}} = \frac{\boldsymbol{U_r - U_j}}{\omega^U}\Big|_{j\neq r}$$

where ω^n and ω^U represent the time lags in firms' and workers' migrations. At the long term equilibrium:

$$\begin{cases} r_1 = r_2 \\ U_1(x) = U_2(x)\ \forall x \end{cases}$$

Differences in living costs matter to households when moving for a new job because they consume in the region where they work. However, living costs do not matter when making a capital investment decision, because they consume their income in their region of residence, which need not be the region where their capital is invested. Therefore, labor mobility is driven by workers' real wage, which includes market prices and urban costs, whereas capital mobility is governed by its nominal rate of return.

3. THE SPATIAL EQUILIBRIUM

3.1 Industries' and households' location

For the sake of simplicity, we assume that the two regions differ only by their number of firms and their number of workers. Therefore, the symmetric configuration, where $L_1 = L_2$ and $n_1 = n_2$, appears to always be a spatial equilibrium. Following Geroliminis and Daganzo (2008), we assume that the average speed of commuting at distance x from the CBD has the following expression:

$$\boldsymbol{v_r}(x) = \frac{v_0}{1 + \left(\frac{\boldsymbol{L_r}}{\boldsymbol{\kappa_r}}\right)^{\alpha}}$$

where v_0 is the average speed without congestion, α is a parameter that captures the decline of the average speed with respect to the number of users of the road network, and $\boldsymbol{\kappa_r}$ is the infrastructure capacity of the urban road network of the region r. The average commuting speed does not vary much when $\boldsymbol{L_r} \ll \boldsymbol{\kappa_r}$, but it drops dramatically when congestion occurs, i.e., when $\boldsymbol{L_r} \sim \boldsymbol{\kappa_r}$. An increase of the population in one region decreases the average commuting speed and the accessibility of workers. Congestion thus acts as a dispersion force. We assume that the average speed without congestion, v_0, is constant within each region.

Considering the fact that the utility of a given worker is constant throughout the city, we obtain the indirect utility as:[4]

[4] The indirect utility is an average utility between employed and unemployed workers living at distance *x* from the CBD, assuming that unemployment rate is constant throughout the city.

Proposition 1 :

$$(9) \qquad \widetilde{\boldsymbol{U}_{\boldsymbol{r}}}(x) = \left(\frac{\boldsymbol{w_r S_r} + \frac{1}{2}\Sigma_{k=1}^{2}\chi \boldsymbol{r_k}\,\boldsymbol{n_k}}{\boldsymbol{P_r}}\right)^{1-\delta^a-\delta^H} \frac{\left(\int_X \frac{\boldsymbol{H_r}(x)}{x^{\frac{\delta^a}{\delta^H}}}\,dx\right)^{\delta^H}}{\boldsymbol{L_r}^{1-\delta^a}} \left(\frac{v_0 T}{1+\left(\frac{\boldsymbol{L_r}}{\boldsymbol{\kappa_r}}\right)^{\alpha}}\right)^{\delta^a}$$

Proof 1: *See the Appendix*

Average utility increases with housing supply, but the repartition of dwellings plays a role. A dwelling that is built further away from the CBD must contribute more to overall utility to compensate for the loss of utility associated with a longer commuting trip. The weight of a given dwelling in the utility function depends thus on the relative share of income workers allocate to housing, δ^H, and to commuting, δ^a. The rent per surface unit, $\boldsymbol{R_r}(x)$, is then equal to:[5]

$$\boldsymbol{R_r}(x) = \frac{\delta^H}{1-\delta^a-\delta^H}\frac{\boldsymbol{w_r L_r}(x)x^{-\frac{\delta^a}{\delta^H}}}{\int_X \boldsymbol{H_r}(x)x^{-\frac{\delta^a}{\delta^H}}\,dx}$$

A population increase at distance x from the CBD will enhance the rent per surface unit, as housing supply is set. Following Equation (4), household consumption of housing surface decreases. Fiercer housing competition acts thus as a dispersion force.

3.2 First case: Population set

To begin, we assume that the population is set evenly between the two regions, namely $L_1 = L_2$. Only the firms can migrate from one region to another. This extreme case could refer to two regions with borders or to a case where firms migrate significantly faster than workers.

3.2.1 Study around the symmetric configuration β = 1/2

We focus first on the symmetric configuration, $n_1 = n_2$, which is a spatial equilibrium. To study its stability, we derive the elasticity of the capital revenue in one region with respect to the number of firms in that region. Totally differentiating $\boldsymbol{r_r}$ and evaluating the resulting expression at $\beta = \frac{1}{2}$, we obtain:

$$\frac{d\boldsymbol{r_r}}{\boldsymbol{r_r}} = \frac{d\boldsymbol{p_r}}{\boldsymbol{p_r}} + \frac{d\boldsymbol{q_r}}{\boldsymbol{q_r}}, r = 1,2$$

As is standard in the literature (Fujita et al., 2001), let

$$Z = \frac{1-\tau^{1-\epsilon}}{1+\tau^{1-\epsilon}}$$

[5] $\boldsymbol{R_r}(x)$ is the bid rent, which is defined as the maximum rent per unit of land that an household can pay for residing at distance x while enjoying the average utility.

where Z is an index of trade cost, with values between 0 and 1. If trade is perfectly costless, $\tau = 1$, Z takes the value 0; if trade is impossible, Z takes the value 1. Then, Equations (5), (3), and (6) imply:

$$\frac{d\boldsymbol{p_r}}{\boldsymbol{p_r}} = \frac{d\boldsymbol{w_r}}{\boldsymbol{w_r}}$$

$$(1-\epsilon)\frac{d\boldsymbol{P_r}}{\boldsymbol{P_r}} = Z\left((1-\epsilon)\frac{d\boldsymbol{p_r}}{\boldsymbol{p_r}} + \frac{d\boldsymbol{n_r}}{\boldsymbol{n_r}}\right)$$

$$\frac{d\boldsymbol{w_r}}{\boldsymbol{w_r}} = \frac{\sigma\frac{\boldsymbol{S_r}}{\boldsymbol{L_r}}}{1-\frac{\boldsymbol{S_r}}{\boldsymbol{L_r}}}\frac{d\boldsymbol{S_r}}{\boldsymbol{S_r}}$$

Similarly, differentiating Equations (7) and (8) yields:

$$\frac{d\boldsymbol{S_r}}{\boldsymbol{S_r}} = \frac{d\boldsymbol{n_r}}{\boldsymbol{n_r}} + \frac{d\boldsymbol{q_r}}{\boldsymbol{q_r}}$$

$$\frac{d\boldsymbol{q_r}}{\boldsymbol{q_r}} = -\epsilon\frac{d\boldsymbol{p_r}}{\boldsymbol{p_r}} + Z\frac{\epsilon-1}{\epsilon}\left(\frac{d\boldsymbol{w_r}}{\boldsymbol{w_r}} + \frac{d\boldsymbol{S_r}}{\boldsymbol{S_r}}\right) + Z(\epsilon-1)\frac{d\boldsymbol{P_r}}{\boldsymbol{P_r}}$$

Solving the six equations above, we derive the elasticity of capital revenue at $n_1 = n_2$:

(10) $$\frac{\boldsymbol{n_r}}{\boldsymbol{r_r}}\frac{d\boldsymbol{r_r}}{d\boldsymbol{n_r}}\Big|_{\beta=\frac{1}{2}} = \frac{\boldsymbol{A_r}(1-\epsilon)+Z\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1)-Z^2\left(1+\boldsymbol{A_r}(2-\epsilon)\right)}{1+\boldsymbol{A_r}\epsilon-Z\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1)+Z^2\boldsymbol{A_r}(1-\epsilon)}$$

where

(11) $$\boldsymbol{A_r} = \frac{\sigma\frac{\boldsymbol{S_r}}{\boldsymbol{L_r}}}{1-\frac{\boldsymbol{S_r}}{\boldsymbol{L_r}}}$$

$\boldsymbol{A_r}$ represents the state of the labor market. A small $\boldsymbol{A_r}$ implies a loose labor market, whereas a tight labor market induces a large $\boldsymbol{A_r}$.

Proposition 2: *If* $\boldsymbol{A_r} < A_d^* = (2\epsilon+1) - \frac{\sqrt{(2\epsilon+1)^2-\left(1-4\frac{2-\epsilon}{\epsilon-1}\right)}}{1-4\frac{2-\epsilon}{\epsilon-1}}$ *, then there exist two break points given by* Z_1^d *and* Z_2^d*,*

$$Z_1^d = \frac{\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1) - \sqrt{\left(\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1)\right)^2 - 4\boldsymbol{A_r}(\epsilon-1)\left(1+\boldsymbol{A_r}(2-\epsilon)\right)}}{2\left(1+\boldsymbol{A_r}(2-\epsilon)\right)}$$

$$Z_2^d = \frac{\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1) + \sqrt{\left(\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1)\right)^2 - 4\boldsymbol{A_r}(\epsilon-1)\left(1+\boldsymbol{A_r}(2-\epsilon)\right)}}{2\left(1+\boldsymbol{A_r}(2-\epsilon)\right)}$$

and the symmetric configuration is unstable if and only if $\boldsymbol{A_r} < A_d^*$ *and* $Z \in [Z_1^d, Z_2^d]$*.*

Proof 2: *See the Appendix*

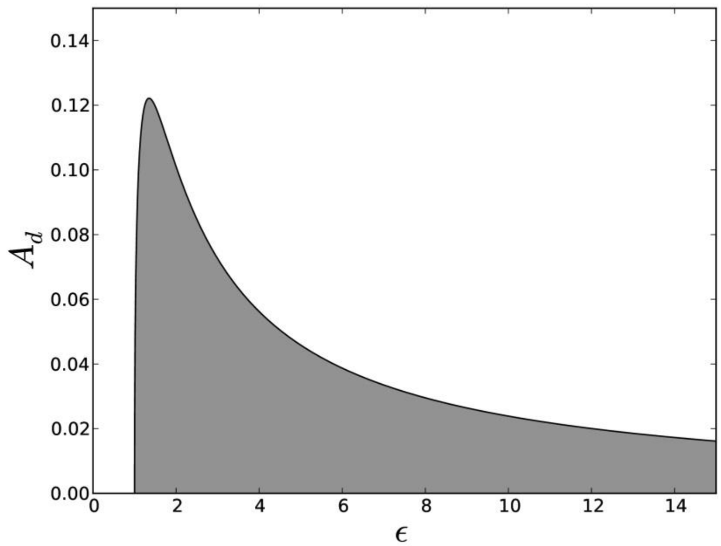
The stability of the symmetric configuration depends on the interplay of agglomeration and dispersion forces. Firms tend to agglomerate because they look for an access to the greater market (*Home Market Effect*). When a firm moves from one region to another, it enhances the employment rate of the region of destination, while it lowers the employment rate of the region of departure. Thus, the income of the region of destination grows for two reasons: the number of employed workers increases and nominal wages rise due to competition in the labor market. The exact inverse mechanism occurs in the region of departure. However, at the same time, the arrival of a new firm makes the competition on the local labor and product markets fiercer. This competition acts as a dispersion force (*Crowding-out Effect*). Therefore, a slight increase in the number of firms in one region will intensify both forces.

The magnitudes of the agglomeration and dispersion forces depend on different parameters. On the one hand, dispersion forces, which rely on competition in labor and product markets, are significant when the local labor market is tight—namely when $\boldsymbol{A_r}$ is relatively large, and when trade costs are low, namely when Z is relatively small. A slight increase in the employment rate raises wages and thus prices, and low trade costs enable firms from the other region to compete in the local product market. On the other hand, agglomeration forces are significant when most of the income generated by the new entrant is spent locally and when firms can compete in the two markets. This occurs when trade costs are low and when the local labor market is loose to prevent competition from the other region.

Proposition 2 shows two cases, depending on the values of $\boldsymbol{A_r}$ and Z. If $\boldsymbol{A_r} > A_d^*$, namely if the unemployment rate is sufficiently low, then dispersion forces overpass agglomeration forces. There is no benefit for a firm to leave a region to move to another one. The slope of the wage curve around a low unemployment rate is very steep, and a weak increase in the number of employed workers will then sharply raise the wage. The arrival of an extra firm in a given region reduces the unemployment rate and increases the wage. This increase drives up the price of varieties produced in that region and the firm's market share decreases. On the contrary, if $\boldsymbol{A_r} < A_d^*$, namely if the unemployment rate is high enough, firms can afford to pay higher wages if they have good access to a larger market, as the rise of wages is weak. If trade costs are less extreme, namely when $Z \in [Z_1^d, Z_2^d]$, firms benefit from all the income generated by the home manufacturing employment and can still compete on the other region's market. However, when trade costs are too low ($Z < Z_1^d$) or too high ($Z > Z_2^d$), the agglomeration benefits cannot offset the increase in production costs. When trade costs are too high, the firms cannot compete in the other region's market, and the competition in the region where the number of firms grew is fiercer. The capital nominal rate of return thus diminishes. If trade costs are too low, firms of the two regions access pretty much the same market that makes the quantity of products sold more reliant on production costs. Firms tend then to scatter to release tensions on the labor market.

The value A_d^* which delimits the case where agglomeration occurs when trade costs are between Z_1^d and Z_2^d to the case where dispersion is always a stable equilibrium is plotted on Figure 1. Below this value, the wage curve is rather flat, and the labor market dispersion force can be offset by the agglomeration benefits. A_d^* increases sharply at first with ϵ, but then decreases. When ϵ is close to 1, varieties are not substitutable. The overall budget devoted to can be offset by the agglomeration benefits. A_d^* increases sharply at first with ϵ, but then

Figure 1: Values of A_r for Which Agglomeration Occurs (Denoted A_d^*)



decreases. When ϵ is close to 1, varieties are not substitutable. The overall budget devoted to each variety is fixed, and firms just try to minimize the trade costs: dispersion is therefore always a stable equilibrium. But then, when ϵ increases, varieties become more and more substitutable. When ϵ is sufficiently small, substitution between varieties is weak, and firms favor greater market access than low production costs. On the contrary, when ϵ is sufficiently high, firms tend to reduce their product price even if they downsize their potential market access.

The state of the labor market in each region plays an important role in the stability of the symmetric configuration. If, in the two regions, the wage is inelastic with respect to the unemployment rate (namely $\sigma = 0$), then there exists only one break point, $Z^d = {}^{(\epsilon - 1)}/_{\epsilon}$. At high trade costs ($Z > Z^d$), the symmetric configuration is always stable, whereas at low trade costs ($Z < Z^d$), the symmetric configuration is always unstable. This result is in line with results from the Core-Periphery model (Krugman, 1991). If wages do not depend on the unemployment rate in the two regions, firms benefit from agglomerating when trade costs are low, as they have access to the greater market without enhancing their production costs.

3.2.2 *Study around the full agglomeration β = 1*

Let us focus now on the full agglomeration configuration. We assume that all firms are gathered in region 1. Population is still evenly divided between the two regions.

Proposition 3: *If* $A_1 < A_a^* = \sigma\left(\frac{\epsilon^2}{2\epsilon-1}\right)^{\frac{1}{2\sigma(\epsilon-1)}}$ *, then there exist two break points given by* Z_1^a *and* Z_2^a*,*

$$Z_1^a = \frac{\frac{\epsilon-1}{\epsilon}\left(\frac{A_1}{\sigma}+1\right)^{\sigma(\epsilon-1)} - \sqrt{1-\left(\frac{A_1}{\sigma}+1\right)^{2\sigma(\epsilon-1)\frac{2\epsilon-1}{\epsilon^2}}}}{1+\left(\frac{A_1}{\sigma}+1\right)^{\sigma(\epsilon-1)}}$$

$$Z_2^a = \frac{\frac{\epsilon - 1}{\epsilon}\left(\frac{A_1}{\sigma}+1\right)^{\sigma\,(\epsilon-1)} + \sqrt{1-\left(\frac{A_1}{\sigma}+1\right)^{2\sigma(\epsilon-1)\frac{2\epsilon-1}{\epsilon^2}}}}{1+\left(\frac{A_1}{\sigma}+1\right)^{\sigma(\epsilon-1)}}$$

and the full agglomeration is a stable equilibrium if and only if $A_1 < A_a^*$ *and* $Z \in [Z_1^a, Z_2^a]$.

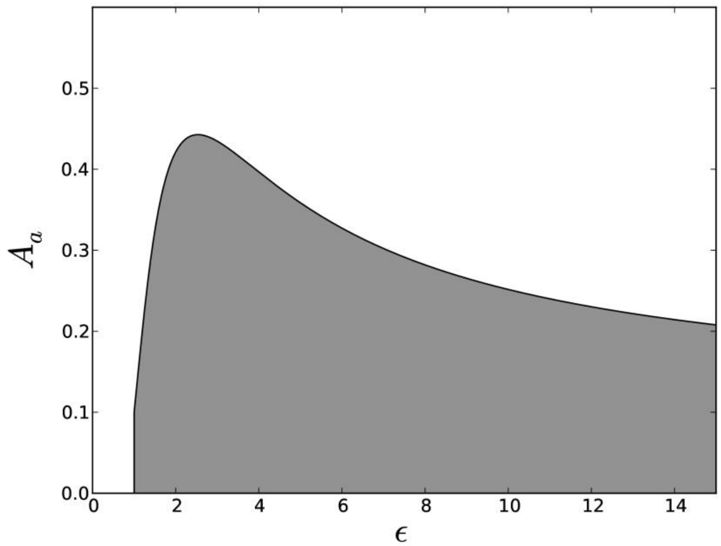
Proof 3: *See the Appendix*

As in the symmetric configuration, the stability of full agglomeration depends on the interplay of agglomeration and dispersion forces. Proposition 3 shows that the intensity of these forces relies on the local labor market, A_1, and the trade cost values, Z. If tensions on the labor market are strong, namely if $A_1 > A_a^*$, then full agglomeration is never an equilibrium. Firms tend to relax competition on the labor market in order to reduce their production costs and enhance their market shares. On the contrary, if the local labor market is not too tight, namely if wages are relatively low, then full agglomeration can be an equilibrium for some specific values of trade costs. As in the symmetric configuration case, if trade costs are not extreme, firms benefit from all the income generated by the home manufacturing employment and can still compete in the other region's market. Full agglomeration then occurs. However, when trade costs are too low or too high, firms have incentives to move to the other region where wages are lower. Indeed, in these cases, production costs in the region where all firms are settled are too high and exceed agglomeration benefits. When trade costs are too high, firms cannot compete in the other region's market, and the first firm that moves to the other region will supply all of the local demand. If trade costs are too low, demand does not depend as much on trade costs but on production costs, and firms tend to reduce the wages they pay to workers. In those two cases, moving from the crowded region to the empty region will enhance capital nominal rates of return. The labor market acts thus as a dispersion force.

The value, A_a^*, beyond which full agglomeration is always unstable is plotted on Figure 2. Below this value, the wage curve is flat enough to push firms to agglomerate completely when trade costs are between Z_1^a and Z_1^b. These are the same mechanisms as when firms are evenly spread across the two regions appear. When ϵ is close to 1, firms want to minimize trade costs because varieties are not substitutable and the demand of each variety is then fixed. Full agglomeration is therefore never a stable equilibrium. When ϵ increases, varieties become more and more substitutable. The demand for each variety depends more and more on its price. When ϵ is sufficiently small, then substitution between varieties is weak and the benefits from a greater market are relatively important. Agglomeration can thus occur even with relatively high wages. Those benefits weaken when ϵ grows, and firms tend to reduce the production price to keep a correct market share.

As for the stability of the symmetric configuration, the state of the labor market in each region plays an important role in the stability of the full agglomeration configuration. If, in the two regions, the wage is inelastic with respect to the unemployment rate (namely σ = 0), then there exists only one sustainable point, $Z^a = {}^{(\epsilon-1)}/_{\epsilon}$. At high trade costs ($Z > Z^a$), the full

Figure 2: Values of A_1 for Which Agglomeration Occurs (Denoted A_a^*)



agglomeration configuration is never stable, whereas at low trade costs ($Z < Z^a$), the full agglomeration configuration is always stable, for the same reasons as for the symmetric configuration. If the wages do not depend on the unemployment rate in the two regions, firms benefit from agglomerating when trade costs are low, as they have access to the greater market without enhancing their production costs. This result is in line with results from the Core-Periphery model (Krugman, 1991).

3.2.3 Bifurcation diagrams

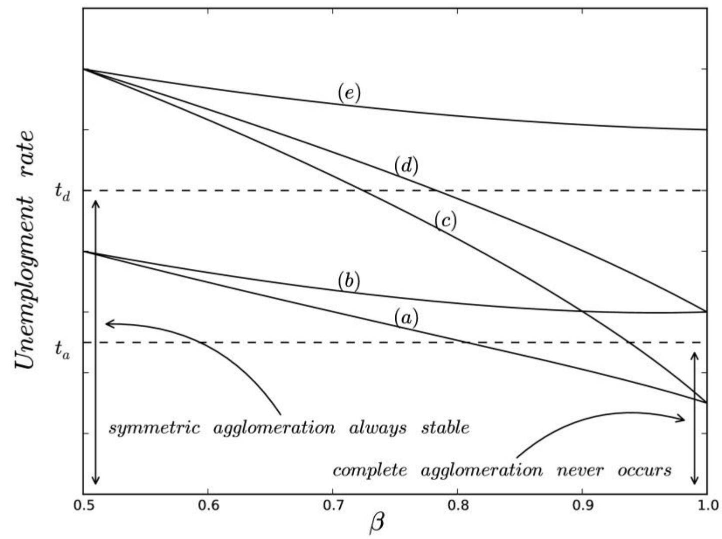
The analysis has dealt with two extreme configurations so far, when firms are equally divided up between the two regions and when all firms are gathered in one region. It has shown that the stability of those two configurations depends highly on the labor market, especially on the unemployment rate which sets the wage and is pictured by $\boldsymbol{A_r}$, and on the trade costs Z. Let us now focus on intermediate configurations in which firms are partially agglomerate in one region. From propositions 2 and 3, we can deduce the following proposition.

Proposition 4 : $\forall\, \epsilon, A_d^* < A_a^*$.

Proof 4: *See the Appendix*

Let t_d be the unemployment rate such that if $1 - \frac{S_r}{L_r} < t_d$, the symmetric configuration is always a stable equilibrium; and let t_a be the unemployment rate such that if $1 - \frac{S_r}{L_r} > t_a$, full agglomeration occurs. Then, proposition 4 asserts that $t_d > t_a$. The above sections state that agglomeration forces are strong when the local labor market is loose, while dispersion forces are strong when the local labor market is tight. Besides, the agglomeration of firms in a given region enhances both forces, but agglomeration forces increase more than dispersion forces with a rise in the number of firms. Thus, agglomeration forces overpass dispersion forces at lower values of the unemployment rate in the full agglomeration configuration than in the symmetric configuration, namely $A_d^* < A_a^*$.

Figure 3: Different Unemployment Rate Trajectories in Region 1



The unemployment rate in a given region is assumed to decrease with the number of firms settled in that region (Francis, 2003).[6] From previous sections, several cases can then be distinguished, depending on the stability of the symmetric and full agglomeration configurations. We study the cases *(a)*, *(b)*, *(c)*, *(d)*, and *(e)* plotted in Figure 3.[7]

In Figure 4, the nominal rate of return for capital between the two regions is plotted versus the number of firms in cases *(a)* and *(b)* for three particular values of trade costs. Stable equilibria are displayed with a black mark, whereas unstable ones are displayed with blank marks. In cases *(a)* and *(b)*, when firms are evenly split between the two regions, unemployment is low, i.e. $\boldsymbol{A_r} > A_d^*$. As seen above, the slope of the wage curve around a low unemployment rate is steep and prevents firms from agglomerating further. Around the symmetric configuration, agglomeration economies cannot offset dispersion effects. The symmetric configuration is always stable. In case *(a)*, we suppose that the unemployment rate decreases sufficiently to prevent scale economies to overpass dispersion effects due to a high wage ($\boldsymbol{A_r} > A_a^*$) when firms are totally gathered. The unique equilibrium is the symmetric configuration, regardless of the trade costs. On the contrary, in case *(b)*, we assume that the unemployment rate decreases slowly with the number of firms and $\boldsymbol{A_r} < A_a^*$ when all firms are gathered in one region. Then, for some values of trade costs (between Z_1^a and Z_2^a), the full agglomeration configuration is stable. Scale economies countervail the dispersion effect as the wage is not high enough to reduce the capital nominal rate of return. When firms agglomerate, income in the region where they settle increases and they benefit from better access to the market. Therefore, there exists a spatial distribution which is neither the full agglomeration configuration nor the symmetric configuration for which dispersion and agglomeration forces are exactly equal. This distribution is nonetheless unstable,

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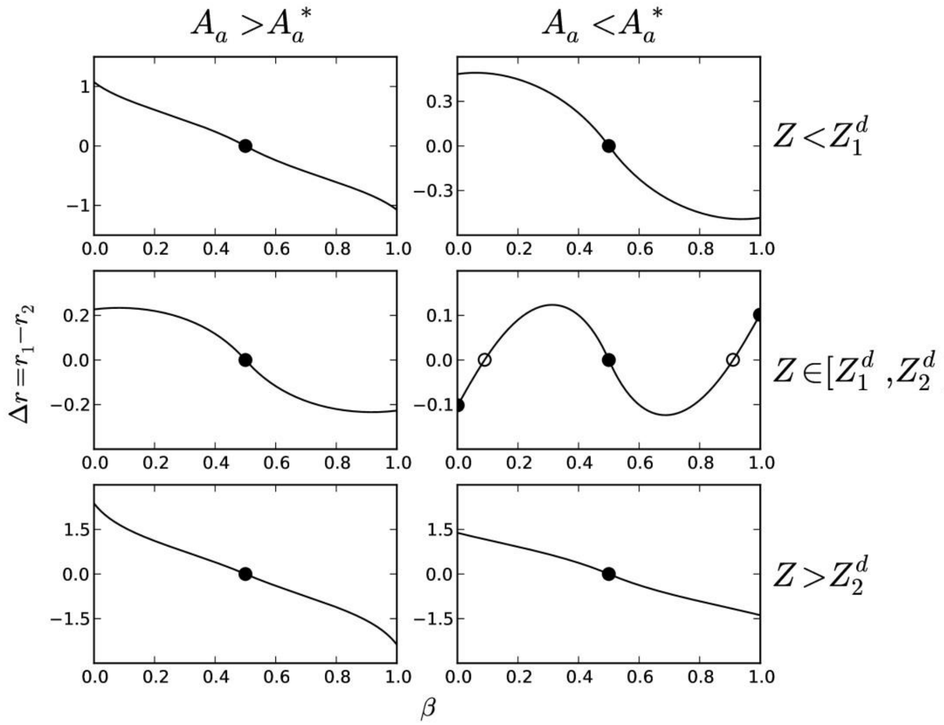
[6] This assumption is in line with empirical facts. For a given population, an increase in the number of firms in one region leads to a growth of employment in that region.

[7] The unemployment rate trajectories are built by assuming an average unemployment rate between the two regions, which varies with the repartition of the firms.

Figure 4: Capital Revenue Difference between the Two Regions in Cases *(a)* and *(b)*

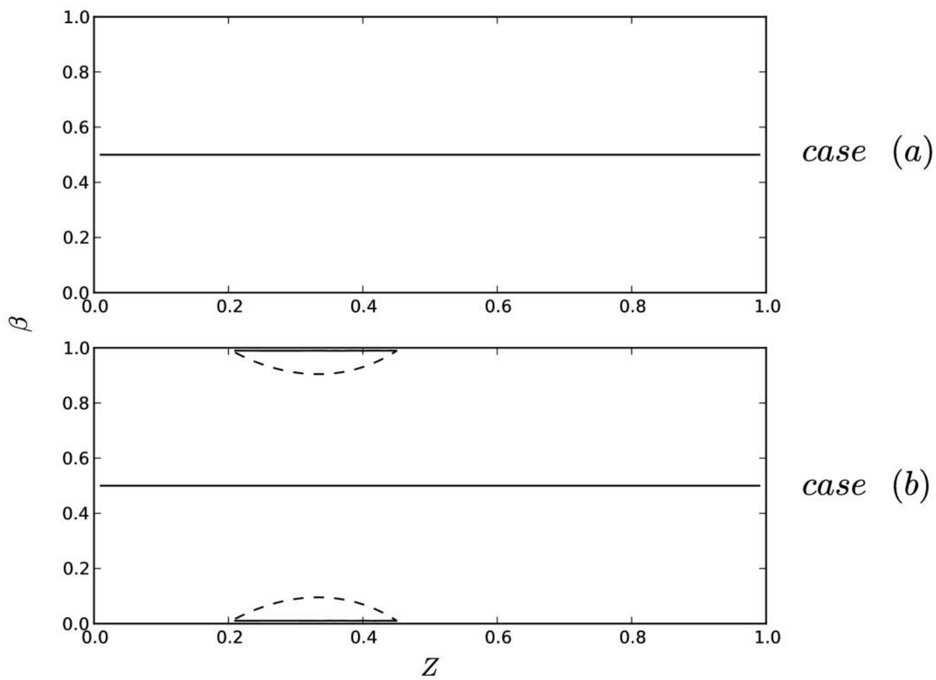


as shown on Figure 4. Several equilibria then appear when trade costs are intermediate. In the symmetric configuration, agglomeration forces are smaller than dispersion forces, whereas in the full agglomeration configuration, agglomeration forces are larger than dispersion forces. These two configurations are thus stable. As firms gradually gather in one region, agglomeration forces increases with respect to dispersion forces. There is thus a configuration in which agglomeration forces exactly balance dispersion forces. This configuration is also an equilibrium, but it is unstable, as a small increase of firms in the region where most firms are gathered will enhance capital rates of return and foster agglomeration.

Figure 5 shows how the equilibria vary with trade costs when $\boldsymbol{A_r} > A_d^*$ in the symmetric configuration. Two cases for the full agglomeration configuration are presented: case *(a)* when $\boldsymbol{A_r} > A_a^*$ on the top and case *(b)* when $\boldsymbol{A_r} < A_a^*$ on the bottom. Solid lines indicate stable equilibria, broken lines unstable. The configuration when the population is evenly divided between the two regions is always a stable equilibrium. But, for case *(b)*, the full agglomeration is also a stable equilibrium when trade costs are intermediate. At high and low trade costs, competition between firms is too fierce and firms tend to scatter to relax competition.

In cases *(c)*, *(d)*, and *(e)*, unlike in cases *(a)* and *(b)*, there exist values of trade costs for which the symmetric configuration is unstable. When firms are evenly split between the two regions, the unemployment rate is relatively high in both regions and the rise of the wage due to the relocation of a firm from one region to another is relatively limited. In the region where the firm settles, the income increases and so does the demand for local goods. As seen in section 3.2.1, if trade costs are neither too high nor too low, firms benefit from all the income generated by locally higher wages and can still compete in the other region's market. Firms tend to

Figure 5: Bifurcation Diagrams in Cases *(a)* and *(b)*



agglomerate in the same region and the symmetric configuration is unstable. Nevertheless, if the employment rate decreases significantly with the number of firms, the dispersion forces due to increasing wages can, at some point, overpass the agglomeration forces. Tensions in the labor market thus prevent further agglomeration. Figure 6 shows the variations of the capital rate of return with the number of firms for three particular values of trade costs. For intermediate trade costs, each case exhibits three spatial equilibria. The symmetric configuration is unstable, but the two other equilibria are stable. In case *(c)*, stable equilibria occur when agglomeration is partial, whereas in case *(d)* and *(e)*, full agglomeration is stable. In the two latter cases, *(d)* and *(e)*, agglomeration forces are always larger than dispersion forces for the intermediate value of trade costs, regardless of the number of firms in the regions. In these cases, the unemployment rate decreases more slowly with the number of firms than in case *(c)*.

In Figure 7, equilibria in the three cases are plotted versus trade costs. Solid lines indicate stable equilibria, broken lines unstable. In case *(d)*, full agglomeration is stable when the symmetric configuration is unstable, namely when $Z_1^a > Z_1^d$ and $Z_2^a < Z_2^d$. This means that there exist equilibria for specific trade values ($Z \in [Z_1^d, Z_1^a] \cup [Z_2^a, Z_2^d]$) which are neither the symmetric configuration nor the full agglomeration configuration. For those values, the unemployment rate is not low enough to enable full agglomeration, as in case *(c)*. In case *(e)*, the only stable equilibria are the symmetric configuration and the full agglomeration configuration. Once agglomeration forces outweigh dispersion forces around the symmetric configuration, firms have no impediment to further agglomeration. We thus have $Z_1^a < Z_1^d$ and $Z_2^a > Z_2^d$.

Figure 6: Capital Revenue Difference between the Two Regions in Cases *(c)*, *(d)*, and *(e)*

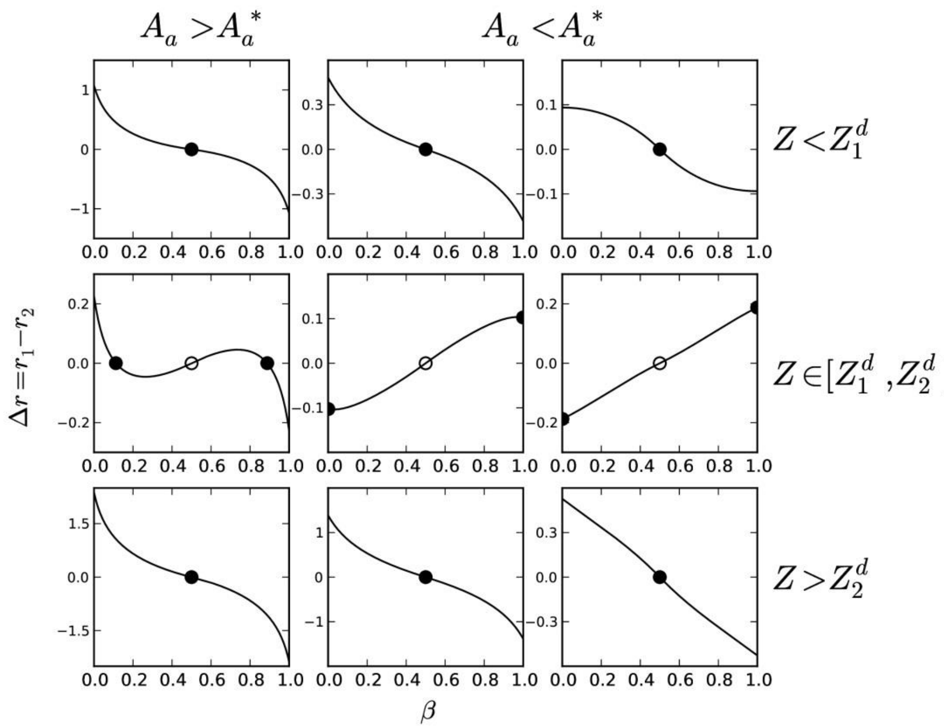
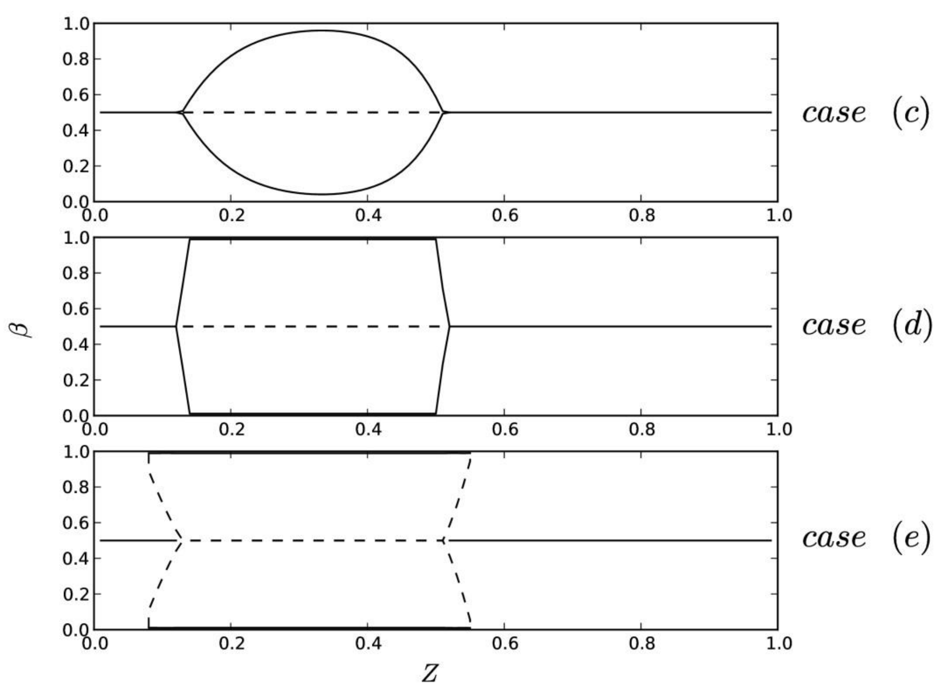


Figure 7: Bifurcation Diagrams in Cases *(c)*, *(d)*, and *(e)*



Firms are attracted by a larger market as it enhances the quantity of product sold and the capital rate of return. But when more firms locate within the same region, local competition on the product and labor markets is intensified and profits are depressed. The dispersive force finds thus its origin in each firm's desire to relax competition on product and labor markets by moving away from competitors. The interplay between the market-access effect and the market-crowding effect shapes the space economy for firms. The intensity of these two effects varies with the level of trade costs and the level of unemployment. When economic integration gets deeper, the intensity of the agglomeration force increases. A higher degree of integration makes exports to the small market easier, which allows firms to exploit more intensively their scale economies. On the other hand, the deepening of integration reduces the advantages associated with geographical isolation in the small market where there is less competition. These two effects push towards more agglomeration of the manufacturing sector. However, the labor market can cancel scale economies if unemployment is much lower in the region where agglomeration occurs and wages are too high, scale economies disappear. In that case, firms tend to scatter and look for smaller production costs and less competition. The results of this section are in line with the New Economic Geography result, the so-called *U-shaped* result. At low and high trade costs, dispersion occurs, while at intermediate trade costs, agglomeration occurs for some value of the parameter $\boldsymbol{A_r}$, which portrays the state of the labor market. Then, the movement of population is expected to play an important role, as people can release tensions on the labor market and favor agglomeration.

3.3 Second case: Firms set

We assume now that the firms are divided up evenly between the two regions, namely $n_1 = n_2$. Workers can migrate from one region to the other, while firms stay in the same region. This extreme case could refer to a case where workers migrate significantly faster than firms.

3.3.1 Study around the symmetric configuration λ = 1/2

We start by focusing on the symmetric configuration, where $L_1 = L_2$. As seen above, this configuration is always a stable equilibrium. To study its stability, we derive the elasticity of indirect utility in one region with respect to the number of workers in that region. Totally differentiating $\boldsymbol{A_r}(x)$ and evaluating the resulting expression at $\lambda = 1/2$, we obtain:

$$(12)\ \frac{d\boldsymbol{U_r}}{\boldsymbol{U_r}} = (1-\delta^m-\delta^H)\left(\frac{\epsilon-1}{\epsilon}\left(\frac{d\boldsymbol{w_r}}{\boldsymbol{w_r}}+\frac{d\boldsymbol{S_r}}{\boldsymbol{S_r}}\right)-\frac{d\boldsymbol{P_r}}{\boldsymbol{P_r}}-\frac{d\boldsymbol{L_r}}{\boldsymbol{L_r}}\right)-\left(\delta^m\frac{\alpha\left(\frac{\boldsymbol{L_r}}{\boldsymbol{\kappa_r}}\right)^{\alpha}}{1+\left(\frac{\boldsymbol{L_r}}{\boldsymbol{\kappa_r}}\right)^{\alpha}}+\delta^H\right)\frac{d\boldsymbol{L_r}}{\boldsymbol{L_r}}$$

Then, Equations (3), (5), and (6) imply

$$\frac{d\boldsymbol{p_r}}{\boldsymbol{p_r}} = \frac{d\boldsymbol{w_r}}{\boldsymbol{w_r}}$$

$$\frac{d\boldsymbol{P_r}}{\boldsymbol{P_r}} = Z\frac{d\boldsymbol{p_r}}{\boldsymbol{p_r}}$$

$$\frac{d\boldsymbol{w_r}}{\boldsymbol{w_r}} = \boldsymbol{A_r}\left(\frac{d\boldsymbol{S_r}}{\boldsymbol{S_r}}-\frac{d\boldsymbol{L_r}}{\boldsymbol{L_r}}\right)$$

Similarly, differentiating Equations (7) and (8) yields:

$$\frac{d\boldsymbol{S_r}}{\boldsymbol{S_r}} = \frac{d\boldsymbol{q_r}}{\boldsymbol{q_r}}$$

$$\frac{d\boldsymbol{q_r}}{\boldsymbol{q_r}} = -\epsilon\frac{d\boldsymbol{p_r}}{\boldsymbol{p_r}} + Z\frac{\epsilon-1}{\epsilon}\left(\frac{d\boldsymbol{w_r}}{\boldsymbol{w_r}} + \frac{d\boldsymbol{S_r}}{\boldsymbol{S_r}}\right) + Z(\epsilon-1)\frac{d\boldsymbol{P_r}}{\boldsymbol{P_r}}$$

Let

$$\mu = \frac{\delta^m \dfrac{\alpha\left(\dfrac{\boldsymbol{L_r}}{\boldsymbol{\kappa_r}}\right)^\alpha}{1+\left(\dfrac{\boldsymbol{L_r}}{\boldsymbol{\kappa_r}}\right)^\alpha} + \delta^H}{1-\delta^m-\delta^H}$$

Solving the six equations above, we have the elasticity of indirect utility at $\lambda = 1/2$:

(13) $$\frac{\boldsymbol{L_r}}{\boldsymbol{U_r}}\frac{d\boldsymbol{U_r}}{d\boldsymbol{L_r}}\Big|_{\lambda=\frac{1}{2}} = (1-\delta^m-\delta^H)\left(\frac{\left(\frac{(\epsilon-1)(\boldsymbol{A_r}+1)}{\epsilon}-Z\boldsymbol{A_r}\right)\left(\frac{\epsilon-1}{\epsilon}Z-1\right)}{1+\boldsymbol{A_r}\epsilon-Z\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1)+Z^2\boldsymbol{A_r}(1-\epsilon)} - \frac{1}{\epsilon} - \mu\right)$$

Proposition 5: *The symmetric configuration is always a stable equilibrium*

Proof 5: *See the Appendix*

Average utility in each region depends on the mean nominal income, the price index, and the urban costs which include commuting and housing costs. Average utility increases with the mean nominal income but falls with the price index and urban costs. Population agglomeration in one region raises the latter, as it induces congestion and fiercer competition in the housing market. Average dwelling surface consumed by workers and average commuting speeds decrease. Effects of population agglomeration on income and the price index are less obvious. Starting from the symmetric configuration, when a worker moves from one region to another, the region of destination benefits first from a looser labor market. Wages are thus depressed and production prices are reduced. Demand of varieties produced in that region is enhanced and firms hire local workers. Unemployment falls and wages rise. But the employment level of the symmetric configuration is not regained. The price index is thus lower in the region where workers agglomerate. A higher unemployment rate and lower wages also reduce the mean nominal income. In our case, the reduction of the price index does not offset the decline of income and the rise of the urban costs, notably because half of capital revenues yielded by the arrival of new workers are sent to the other region. The redistribution of capital revenues plays an important role in the workers' incentives to move. Another redistribution that one could allow is for the average utility in the region where the workers agglomerate to be bigger than in the other region.[8]

3.3.2 Study around the full agglomeration λ = 1

We now focus on the case of full agglomeration: we assume that all workers are gathered in region 1 ($\lambda = 1$). In region 2, there is no production anymore because there are no workers. We thus have:

(14) $$\frac{U_1}{U_2} \sim_{\lambda\to 1} \left((2\epsilon-1)\tau\right)^{1-\delta^m-\delta^H}\left(\frac{1}{1+\left(\frac{1}{\kappa_1}\right)^\alpha}\right)^{\delta^m}(1-\lambda)^{1-\delta^m}$$

---

[8] This is not the purpose of this paper, but further research should be done on this topic.

Proposition 6: *Full agglomeration is never a spatial equilibrium*

Proof 6: *From (11), we have*

$$\lim_{\lambda \to 1} \frac{U_1}{U_2} = 0 < 1$$

If workers agglomerate entirely in region 1, utility in region 2 becomes much larger than utility in region 1, and the incentives to move from region 1 to region 2 are important. In that case, dispersion forces outweigh considerably agglomeration forces.

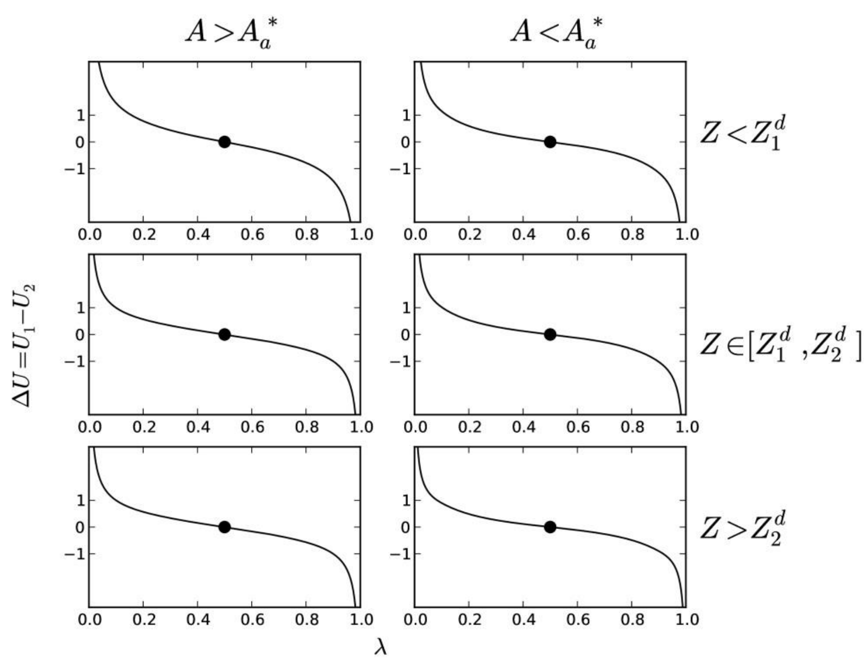
The same mechanisms described above occur here. Full agglomeration considerably heightens urban costs in the region where workers settle and reduces average income as unemployment increases, wages drop, and capital revenues are split between the two regions. In the same time, it lowers the price index. The fall of the price index is weak compared to the dispersion effects due to urban costs and average income.

3.3.3 Bifurcation diagrams

The two sections above show that the symmetric configuration is always a stable equilibrium, while the full agglomeration in one region is never a spatial equilibrium. We now focus on the intermediate configurations, for several values of employment rate ($\boldsymbol{A_r}$) and several values of trade costs (Z). We assume that the employment rate in a given region is constant with the number of workers settled in that region. Figure 8 exhibits how the difference of utility between the two regions varies in that case.

For those cases, there is only one stable equilibrium, which is the symmetric configuration. The decrease of the price index with the agglomeration of workers never heightens the utility as the average income decreases and urban costs rise. As expected considering the utility function, workers avoid agglomeration.

Figure 8: Utility Difference between the Two Regions



3.4 General case

The above two sections deal with one mobile and one immobile factor of production. We consider now that firms and workers are free to migrate wherever they want. The mechanisms highlighted above could be enhanced or reduced.

3.4.1 Study around the symmetric configuration λ = 1/2 and β =1/2

One more time, we focus on the symmetric configuration, where population and firms are set evenly between the two regions, $n_1 = n_2$ and $L_1 = L_2$. This configuration is a spatial equilibrium. To study its stability, we derive the elasticity of the capital rate of return and the indirect average utility in one region with respect to the number of firms and workers in that region. As in the previous sections, totally differentiating $\boldsymbol{r_r}$ and $\boldsymbol{U_r}$, we obtain:

$$\frac{d\boldsymbol{r_r}}{\boldsymbol{r_r}} = \frac{d\boldsymbol{p_r}}{\boldsymbol{p_r}} + \frac{d\boldsymbol{q_r}}{\boldsymbol{q_r}}$$

$$\frac{d\boldsymbol{U_r}}{\boldsymbol{U_r}} = (1-\delta^m-\delta^H)\left(\frac{\epsilon-1}{\epsilon}\left(\frac{d\boldsymbol{w_r}}{\boldsymbol{w_r}} + \frac{d\boldsymbol{S_r}}{\boldsymbol{S_r}}\right) - \frac{d\boldsymbol{P_r}}{\boldsymbol{P_r}} - \frac{d\boldsymbol{L_r}}{\boldsymbol{L_r}}\right) - \left(\delta^m \frac{\left(\alpha\left(\frac{\boldsymbol{L_r}}{\boldsymbol{\kappa_r}}\right)^{\alpha}\right)}{1+\left(\frac{\boldsymbol{L_r}}{\boldsymbol{\kappa_r}}\right)^{\alpha}} + \delta^H\right)\frac{d\boldsymbol{L_r}}{\boldsymbol{L_r}}$$

Then, Equations (1), (2), and (3) imply

$$\frac{d\boldsymbol{p_r}}{\boldsymbol{p_r}} = \frac{d\boldsymbol{w_r}}{\boldsymbol{w_r}}$$

$$(1-\epsilon)\frac{d\boldsymbol{P_r}}{\boldsymbol{P_r}} = Z\left((1-\epsilon)\frac{d\boldsymbol{p_r}}{\boldsymbol{p_r}} + \frac{d\boldsymbol{n_r}}{\boldsymbol{n_r}}\right)$$

$$\frac{d\boldsymbol{w_r}}{\boldsymbol{w_r}} = \boldsymbol{A_r}\left(\frac{d\boldsymbol{S_r}}{\boldsymbol{S_r}} - \frac{d\boldsymbol{L_r}}{\boldsymbol{L_r}}\right)$$

Similarly, differentiating Equations (4) and (5)

$$\frac{d\boldsymbol{S_r}}{\boldsymbol{S_r}} = \frac{d\boldsymbol{n_r}}{\boldsymbol{n_r}} + \frac{d\boldsymbol{q_r}}{\boldsymbol{q_r}}$$

$$\frac{d\boldsymbol{q_r}}{\boldsymbol{q_r}} = -\epsilon\frac{d\boldsymbol{p_r}}{\boldsymbol{p_r}} + Z\frac{\epsilon-1}{\epsilon}\left(\frac{d\boldsymbol{w_r}}{\boldsymbol{w_r}} + \frac{d\boldsymbol{S_r}}{\boldsymbol{S_r}}\right) + Z(\epsilon-1)\frac{d\boldsymbol{P_r}}{\boldsymbol{P_r}}$$

Solving the seven equations above, we have the partial elasticities of capital rate of return and indirect utility at $n_1 = n_2$ and at $L_1 = L_2$:

$$(15)\quad \begin{cases} \dfrac{\boldsymbol{n_r}}{\boldsymbol{r_r}}\dfrac{\delta\boldsymbol{r_r}}{\delta\boldsymbol{n_r}} = \dfrac{\boldsymbol{A_r}(1-\epsilon) + Z\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1) - Z^2\left(1+\boldsymbol{A_r}(2-\epsilon)\right)}{1+\boldsymbol{A_r}\epsilon - Z\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1) + Z^2\boldsymbol{A_r}(1-\epsilon)} \\[2ex] \dfrac{\boldsymbol{L_r}}{\boldsymbol{r_r}}\dfrac{\delta\boldsymbol{r_r}}{\delta\boldsymbol{L_r}} = \dfrac{\boldsymbol{A_r}(\epsilon-1)(1-Z^2)}{1+\boldsymbol{A_r}\epsilon - Z\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1) + Z^2\boldsymbol{A_r}(1-\epsilon)} \end{cases}$$

$$(16)\begin{cases} \dfrac{\boldsymbol{L_r}}{\boldsymbol{U_r}}\dfrac{\delta \boldsymbol{U_r}}{\delta \boldsymbol{L_r}} = (1-\delta^m-\delta^H)\left(\dfrac{\left(\frac{(\epsilon-1)(\boldsymbol{A_r}+1)}{\epsilon}-Z\boldsymbol{A_r}\right)\left(\frac{\epsilon-1}{\epsilon}Z-1\right)}{1+\boldsymbol{A_r}\epsilon-Z\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1)+Z^2\boldsymbol{A_r}(1-\epsilon)}-\dfrac{1}{\epsilon}-\mu\right) \\ \\ \dfrac{\boldsymbol{n_r}}{\boldsymbol{U_r}}\dfrac{\delta \boldsymbol{U_r}}{\delta \boldsymbol{n_r}} = (1-\delta^m-\delta^H)(\dfrac{(\boldsymbol{A_r}+1)\left(\frac{\epsilon-1}{\epsilon}+Z\frac{1}{\epsilon-1}-Z^2\right)}{1+\boldsymbol{A_r}\epsilon-Z\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1)+Z^2\boldsymbol{A_r}(1-\epsilon)} \end{cases}$$

The two first equations of the above Equation sets (15) and (16) were studied in the previous two sections. Let us focus on the two other equations.

Proposition 7:

$$\forall\epsilon, \forall A, \forall Z, \qquad \frac{\boldsymbol{L_r}}{\boldsymbol{r_r}}\frac{\delta \boldsymbol{r_r}}{\delta \boldsymbol{L_r}} > 0$$

$$\frac{\boldsymbol{n_r}}{\boldsymbol{U_r}}\frac{\delta \boldsymbol{U_r}}{\delta \boldsymbol{n_r}} > 0$$

Proof 7: *See the Appendix.*

Around the symmetric configuration, Proposition 7 shows that an increase in the number of firms in one region will raise workers' utility of that region, and an increase in the number of workers will heighten the capital rate of return. A rise of the number of firms in one region will enlarge the number of varieties available and reduce the number of unemployed workers. The price index will fall and the wages will increase. Workers' utilities will thus increase. A rise of the number of workers in one region will extend firms' market access and reduce wages as unemployment grows. The wage lessening will augment the number of products sold, leading to an increase of the capital rate of return. Therefore, if firms or workers have incentives to move from a region to another, this phenomenon will be intensified around the symmetric equilibrium. Workers will follow firms and vice versa.

Proposition 8: *Let C_d be:*

$$C_d = ((1+\mu)(\epsilon-1-\boldsymbol{A_r})+\epsilon\boldsymbol{A_r}(2+\mu))^2+4\epsilon\boldsymbol{A_r}(\epsilon-1)(1+\mu\epsilon)(\boldsymbol{A_r}(\mu\epsilon-1-2\mu) - (1+\mu))$$

If $C_d > 0$, then there exist two break points given by Z_1^{*d} and Z_2^{*d},

$$Z_1^{*d} = \frac{(\boldsymbol{A_r}+1-\epsilon)(1+\mu)-\epsilon\boldsymbol{A_r}(2+\mu)-\sqrt{C_d}}{2\epsilon(\boldsymbol{A_r}(\mu\epsilon-2\mu-1)-(1+\mu))}$$

$$Z_2^{*d} = \frac{(\boldsymbol{A_r}+1-\epsilon)(1+\mu)-\epsilon\boldsymbol{A_r}(2+\mu)+\sqrt{C_d}}{2\epsilon(\boldsymbol{A_r}(\mu\epsilon-2\mu-1)-(1+\mu))}$$

and the symmetric configuration is unstable if and only if $C_d > 0$ and $Z \in [Z_1^{*d}, Z_2^{*d}]$.

Proof 8: *See the Appendix*

The stability of the symmetric configuration depends on the interplay of agglomeration and dispersion forces for both firms and workers. The symmetric configuration is unstable for intermediate trade costs if the labor market is not too tight (low values of $\boldsymbol{A_r}$), urban costs are not too high (low values of μ), and varieties are not too substitutable (low values of ϵ). These results are in line with the previous sections. Interestingly, the symmetric configuration might be unstable even if workers and firms have no incentive to agglomerate in a particular region. For these cases, a small deviation from the symmetric configuration has a snowball effect, workers follow firms and vice-versa. Such a case is studied later (case (2) in Section 3.4.3).

3.4.2 Study around the full agglomeration $\lambda = 1$ and $\beta = 1$

We now focus on the case where firms or workers agglomerate in the same region, Region 1. Let us study the incentives for firms or for workers to leave the region where they are agglomerate.

Proposition 9: *Full agglomeration is never a spatial equilibrium for workers.*

Proof 9: *See the Appendix*

Proposition 9 is in line with the previous results. Full agglomeration of workers considerably intensifies urban costs. Even in the case where workers and firms are located in the same region, the urban costs are not offset by the fall of the price index. Besides, workers compete in the labor market which reduces wages. Income falls with agglomeration, as the capital revenues are equally split between the two regions. Workers thus are better off leaving the region where they are agglomerated.

Considering firms, the stability of the full agglomeration in one region depends on the wage in that region. If unemployment is low, firms tend to benefit from the higher income due to the redistribution of capital revenues and to the relaxation of competition among firms. On the contrary, if wages are low, firms can better exploit their scale economies and benefit from agglomeration benefits. If trade costs are too high, they cannot compete in the market of the other region, and incentives for firms to move to the empty region are high. If trade costs are too low, their access to the market is pretty much the same in each region. Location matters less, and firms tend to limit the labor cost by leaving the region with most of the firms. The population repartition between the two regions plays a role in firms' location choices, as they modify the labor market and thus the wage.

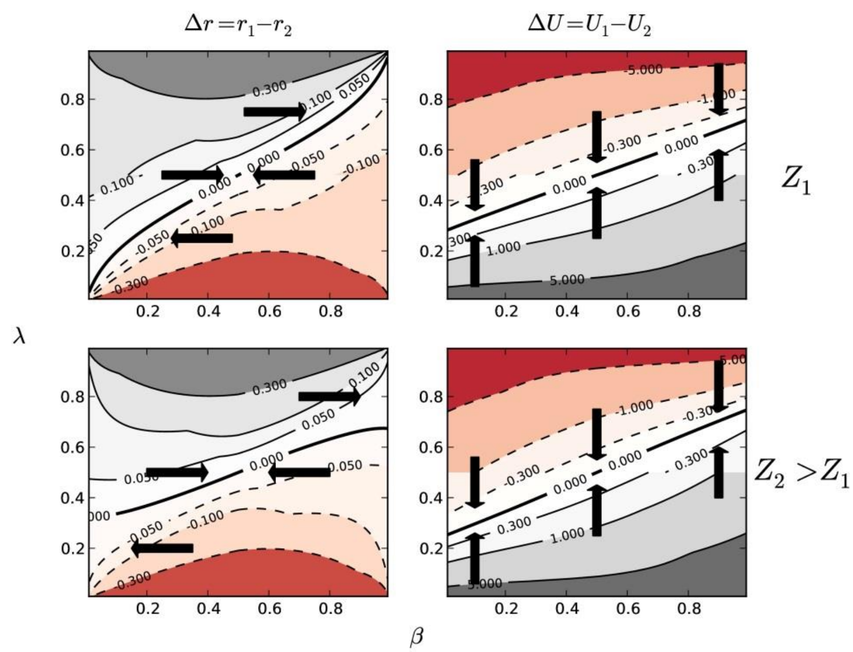
3.4.3 The set of equilibria: Numerical examples

We now consider general configurations. In previous sections, we noticed that when firms and the population move to a new region, local market conditions are affected. On the one hand, given trade costs, the presence of a new competitor intensifies local competition and reduces the local price index. This has a negative impact on demand per firm (*Crowding-out Effect*) and a positive impact on consumer cost of living (*Price Index Effect*). The negative impact on demand per firm is alleviated by the growth of local expenditures as long as part of the income generated by the new entrant is spent locally (*Home Market Effect*). This latter effect is heightened when the labor market is tight and the wages are high.

On the other hand, given trade costs, the presence of a new worker decreases the local competition in the labor market and increases the demand for local products. This has a positive impact on demand per firm and a negative impact on consumer wages. This negative impact is stronger when the labor market is tight, and influences the demand per firm by the drop of local expenditures and production prices which can enhance demand from the other region. Besides, the presence of a new worker increases urban costs, as commuting times get longer and housing prices get higher. The intensities of these mechanisms depend highly on the local labor market, i.e. the unemployment rate, and on the trade costs between the two regions. We study two specific cases, (1) and (2), which differ with regards to their unemployment rates for several values of trade costs. Case (1) is characterized by an averagely tight labor market while case (2) is featured by an averagely loose labor market.

Let us consider first case (1). In Figure 9, the difference of the capital nominal rate of return (on the left) and the difference of the utility (on the right) between the two regions are plotted versus the number of firms and the number of workers in Region 1 for two different values of trade costs, Z_1 and $Z_2 > Z_1$. It appears that the region with the larger number of firms attracts the workers, as the agglomeration of firms reduces the price index through an increase in the number of varieties locally available and pushes wages upward through a crowding effect on the labor market. However, the agglomeration of workers in the same region increases urban costs and decreases wages, and thus hampers their utilities. On the contrary, the most populated

Figure 9: Capital Revenue and Utility Differences between the Two Regions in Case (1)



region appeals to firms: local demand is the highest and production costs are the lowest. But competition on the labor and the goods markets gets fiercer where firms settle. The production costs increase and their market shares fall. Therefore, firms tend to avoid the region with the larger number of firms. The black arrows show the relocation of firms resulting from differences in the capital nominal rate of return and the migration of workers due to differences of utility between the two regions. The thick black line displays the equilibria for the firms and for the workers.

Now, let us consider case (2). In Figure 10, the difference in the capital nominal rate of return (on the left) and the difference in the utility (on the right) between the two regions are plotted versus the number of firms and the number of workers in region 1 for two different values of trade costs. The same incentives that were present in case (1) are at play for the migrations of workers and firms. However, case (2) is featured by an averagely loose labor market, where competition in the labor markets is weaker. Firms can thus have an incentive to settle in the region with the larger number of firms, as they benefit from the rise of local expenditures and can still compete in the other region's market. This effect is fostered by intermediary trade costs, which prevent the region with the larger number of firms from fierce competition from the other region but allows some trade between the two regions, and capital revenues in the region with the larger number of firms can overpass those in the other region (as seen in Figure 10 when $Z = Z_2$). New equilibria thus can occur. The black arrows show the relocation of firms resulting from differences in the capital nominal rate of return and the migration of workers due to differences of utility between the two regions. The thick black line displays the equilibria for workers and firms. As described above, new equilibria occur compare to the case (1).

From Figure 9 and Figure 10, we notice that the evolution of the difference in utility and the capital nominal rate of return against the number of firms and the population settled in one

Figure 10: Capital Revenue and Utility Differences between the Two Regions in Case (2)

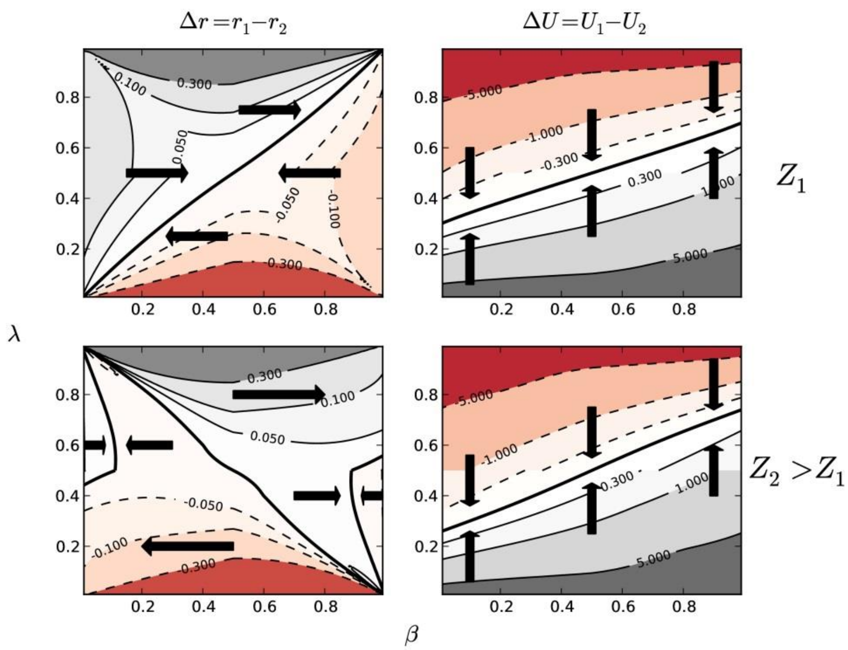
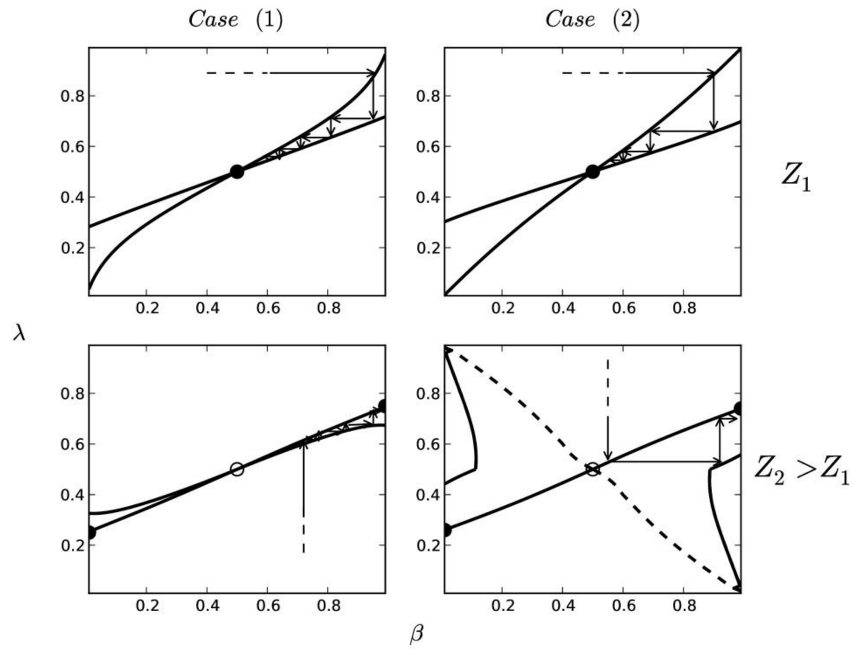


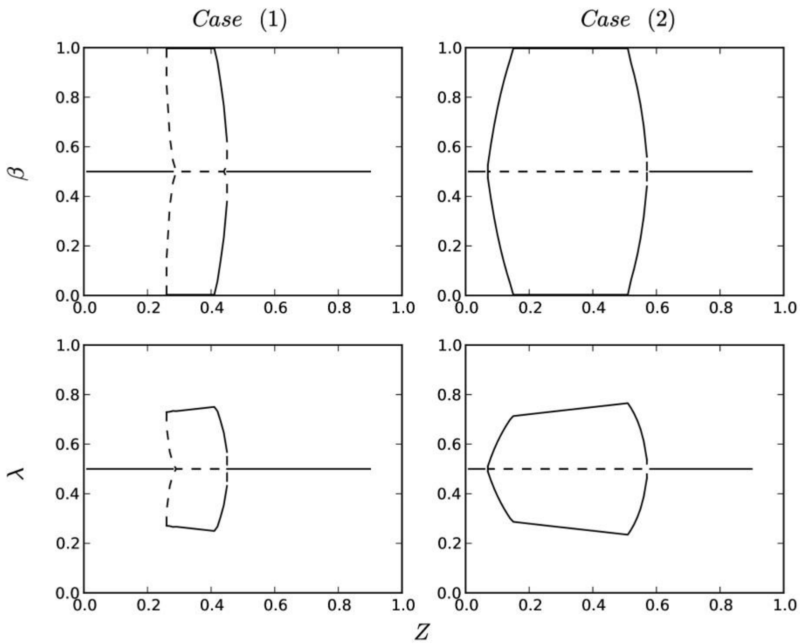
Figure 11: Stable and Unstable Equilibria in One Region



region in cases (1) and (2) for two different values of trade costs. From this evolution, Figure 11 is plotted and displays the stable equilibria with black marks and the unstable equilibria with blank marks. In both cases (1) and (2), when $Z = Z_1$ and capital nominal rates of return in region 1 and region 2 are equal, namely when firms have no incentives to migrate, workers' utility in the region with the most firms is lower. Workers then tend to migrate to the region with fewer firms. They do not want to migrate further when utilities are the same in both regions. Firms then have an incentive to migrate to the less crowded region, and so on. A unique equilibrium appears where firms and workers are evenly split between the two regions. This equilibrium is stable. When $Z = Z_2$, in case (1), the capital nominal rates of return in Region 1 and Region2 are equal for distributions of workers between the two regions that makes the utility in the region with the most firms higher. Workers tend then to migrate to the region with more firms, and firms have the incentive to migrate to the more crowded region. Two stable equilibria appear where one region host all firms but only a fraction of workers. The configuration where firms and workers are evenly split up between the two regions is also an equilibrium, but unstable. In case (2), when $Z = Z_2$, firms tend to agglomerate in one region, regardless of the number of worker in that region. As workers have incentives to settle in the region with the more firms, two equilibria emerge, where all firms are agglomerated in a region with the highest fraction of workers. Such as in case (1), the symmetric configuration is also an equilibrium, but an unstable one.

Figure 12 displays the unstable and stable equilibria versus the trade costs in case (1) and (2) for firms and workers. It appears that, in the two cases, for high trade costs, the only stable equilibria is the symmetric configuration. Firms can compete only in the region where they settle, and thus give more priority to relax competition between them than to settle in the larger region. They therefore tend to divide up between the two regions. Workers follow firms and also split up between the two regions. For low trade costs, the only stable equilibria is the symmetric configuration. Firms have access to the same market from any region. They favor less competition between themselves, like for high trade costs, and tend to split up between the two

Figure 12: Bifurcation Diagrams in Cases (1) and (2)



regions. Workers do the same. For intermediary trade costs, firms can compete on the market of the other region if their production costs are not too high. They then favor greater access to the region with the largest market (namely the largest total income), even if the resulting location decision increases competition between them. The agglomeration of firms in one region makes the income of that region even larger and the agglomeration effects are enhanced. A region attracts all the firms. Workers, when trade costs are low or high, follow firms. But at some point, the agglomeration of workers raises significantly the urban costs and agglomeration stops. The region with all the firms attracts most of the workers, but not all. In other words, when one region is larger in terms of population, the equilibrium is reached when this region attracts a more than proportional share of firms (*Home Market Effect*). The intensity of the home market effect varies with the level of trade costs. Figure 12 shows the well-known *U-shaped* curve of the New Economic Geography model. It is worth noting that the range of trade costs for which agglomeration occurs depends strongly on the features of the labor market. When the labor market is tight (case 1), agglomeration happens on a smaller range of trade costs than when the labor market is loose (case 2). But the two cases display a bell-shaped curve of spatial development.

4. CONCLUDING REMARKS

This paper has developed a model inspired by the New Economic Geography model that includes an explicit description of local labor markets and their interactions with the trade costs of goods. It appears that the space economy displays a bell-shaped curve of spatial development both for firms and workers. For high and low trade costs, firms and workers tend to split up between the two regions to avoid competition on the product and the labor markets, whereas for intermediate trade costs, they tend to agglomerate. For intermediate trade costs, firms gather completely in the region with the largest market, even if agglomeration enhances competition. However, urban costs prevent population from full agglomeration. By being agglomerated, workers save on trade costs of differentiated products, but bear higher housing and commuting costs. The magnitude of the dispersion forces, linked in particular to competition in labor market, and of the agglomeration forces, linked to access to a larger market, depends on the trade costs and on the features of the labor market.

Therefore, what really matters for the structure of the space economy is not just the level of economic integration, but the interplay between trade costs and the local labor markets. Our model can be used as a building-block to elaborate more complex models calibrated on real urban structures. A possible extension is to allow for heterogeneous agents, in line with recent research on the relevance of heterogeneity in labor economics or international trade. This may throw some light on how the interactions between heterogeneity among people and firms affect the intensity and the existence of agglomeration and dispersion forces. Another extension could be to confront the model outlined in this paper with empirical data. This may help justify the empirical relevance of the agglomeration and dispersion forces described in this paper.

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APPENDIX

Proof 1: From Equations (1), (2), and (3), the utility of a worker living at distance x from the CBD is:

$$\boldsymbol{U_r}(x) = \left(\frac{1-\delta^a-\delta^H}{1-\delta^a}\frac{\boldsymbol{Y_r}(x)}{\boldsymbol{P_r}}\right)^{1-\delta^a-\delta^H} \left(\frac{\boldsymbol{H_r}(x)}{\boldsymbol{L_r}(x)}\right)^{\delta^H} \left(\frac{\boldsymbol{v_r}(x)T}{x}\right)^{\delta^a}$$

We assume that the unemployment rate is constant throughout the city. Then, the income averaged over unemployed and employed workers living at distance x from the CBD ($\widetilde{\boldsymbol{Y_r}}$) does not depend on the distance x and is equal to:

$$\widetilde{\boldsymbol{Y_r}} = \frac{1-\delta^a}{1-\delta^a-\delta^H}\left(\boldsymbol{w_r}\frac{\boldsymbol{S_r}}{\boldsymbol{L_r}} + \frac{1}{2\boldsymbol{L_r}}\sum_{\boldsymbol{k=1}}^{\boldsymbol{2}}\chi\boldsymbol{r_k}\boldsymbol{n_k}\right)$$

Assuming that the equilibrium is reached within the city, we have the indirect utility, averaged over unemployed and employed workers living in region r:

$$\widetilde{\boldsymbol{U_r}}(x) = \left(\frac{1-\delta^a-\delta^H}{1-\delta^a}\frac{\widetilde{\boldsymbol{Y_r}}}{\boldsymbol{P_r}}\right)^{1-\delta^a-\delta^H} \left(\frac{\int_X \frac{\boldsymbol{H_r}(x)}{x^{\frac{\delta^a}{\delta^H}}}\,dx}{\boldsymbol{L_r}}\right)^{\delta^H} \left(\frac{v_0 T}{1+\left(\frac{\boldsymbol{L_r}}{\boldsymbol{\kappa_r}}\right)^{\alpha}}\right)^{\delta^a}$$

Combining the two equations above, we have:

$$\widetilde{\boldsymbol{U_r}}(x) = \left(\frac{\boldsymbol{w_r}\boldsymbol{S_r} + \frac{1}{2}\Sigma_{k=1}^{2}\chi\boldsymbol{r_k}\,\boldsymbol{n_k}}{\boldsymbol{P_r}}\right)^{1-\delta^a-\delta^H} \frac{\left(\int_X \frac{\boldsymbol{H_r}(x)}{x^{\frac{\delta^a}{\delta^H}}}\,dx\right)^{\delta^H}}{\boldsymbol{L_r}^{1-\delta^a}} \left(\frac{v_0 T}{1+\left(\frac{\boldsymbol{L_r}}{\boldsymbol{\kappa_r}}\right)^{\alpha}}\right)^{\delta^a}$$

Proof 2: The symmetric configuration is a stable equilibrium if and only if a growth in the number of firms in one region around the symmetric equilibrium leads to a smaller capital rate of return in the largest region than in the smallest. Namely,

$$\frac{\boldsymbol{n_r}}{\boldsymbol{r_r}}\frac{d\boldsymbol{r_r}}{d\boldsymbol{n_r}}\Big|_{\beta=\frac{1}{2}} < 0$$

From Equation (7), we have:

$$\frac{\boldsymbol{n_r}}{\boldsymbol{r_r}}\frac{d\boldsymbol{r_r}}{d\boldsymbol{n_r}}\Big|_{\beta=\frac{1}{2}} < 0 \Leftrightarrow \frac{\boldsymbol{A_r}(1-\epsilon) + Z\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1) - Z^2\big(1+\boldsymbol{A_r}(2-\epsilon)\big)}{1+\boldsymbol{A_r}\epsilon - Z\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1) + Z^2\boldsymbol{A_r}(1-\epsilon)} < 0$$

We note:

$$D = 1 + \boldsymbol{A_r}\epsilon - Z\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1) + Z^2\boldsymbol{A_r}(1-\epsilon)$$

We thus have,

$$D > 0 \;\Leftrightarrow\; Z \in \;[Z_D^{(1)}, Z_D^{(2)}]$$

where,

$$Z_D^{(1)} = \frac{-\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1) - \sqrt{\left(\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1)\right)^2 + 4\boldsymbol{A_r}(\epsilon-1)(1+\epsilon\,\boldsymbol{A_r})}}{2\boldsymbol{A_r}(\epsilon-1)}$$

$$Z_D^{(2)} = \frac{-\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1) + \sqrt{\left(\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1)\right)^2 + 4\boldsymbol{A_r}(\epsilon-1)(1+\epsilon\,\boldsymbol{A_r})}}{2\boldsymbol{A_r}(\epsilon-1)}$$

However, $\forall\epsilon, \forall\mathbf{A_r}$,

$$\frac{-\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1) - \sqrt{\left(\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1)\right)^2 + 4\boldsymbol{A_r}(\epsilon-1)(1+\epsilon\,\boldsymbol{A_r})}}{2\boldsymbol{A_r}(\epsilon-1)} < 0$$

$$\frac{-\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1) + \sqrt{\left(\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1)\right)^2 + 4\boldsymbol{A_r}(\epsilon-1)(1+\epsilon\boldsymbol{A_r})}}{2\boldsymbol{A_r}(\epsilon-1)} > 1$$

Therefore, as $Z \in [0,1]$, we have $\forall\epsilon, \forall\boldsymbol{A_r}, D > 0$.

We note:

$$N = \boldsymbol{A_r}(1-\epsilon) + Z\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1) - Z^2(1+\boldsymbol{A_r}(2-\epsilon))$$

Considering Δ, the discriminant, we have:

$$\Delta = \left(\frac{\epsilon - 1}{\epsilon}\right)^2 (\boldsymbol{A_r} + 1)^2 - 4\boldsymbol{A_r}(\epsilon - 1)\big(1 + \boldsymbol{A_r}(2 - \epsilon)\big)$$
$$\Delta < 0 \iff \boldsymbol{A_r} \in [A_d^{(1)}, A_d^{(2)}]$$

where,

$$A_d^{(1)} = \frac{(2\epsilon + 1) - \sqrt{(2\epsilon + 1)^2 - \left(1 - 4\frac{2 - \epsilon}{\epsilon - 1}\right)}}{1 - 4\frac{2 - \epsilon}{\epsilon - 1}}$$
$$A_d^{(2)} = \frac{(2\epsilon + 1) + \sqrt{(2\epsilon + 1)^2 - \left(1 - 4\frac{2 - \epsilon}{\epsilon - 1}\right)}}{1 - 4\frac{2 - \epsilon}{\epsilon - 1}}$$

If $\boldsymbol{A_r} > A_d^{(2)}$, N has two roots which are superior to 1. Besides, if $\boldsymbol{A_r} > A_d^{(1)}$, $\Delta < 0$, then N has no roots. In both cases, $\forall Z \in [0,1], N > 0$. If $\boldsymbol{A_r} < A_d^{(1)}$, N has two roots, Z_1^d and Z_2^d, such that:

$$Z_1^d = \frac{\frac{\epsilon - 1}{\epsilon}(\boldsymbol{A_r} + 1) - \sqrt{\left(\frac{\epsilon - 1}{\epsilon}(\boldsymbol{A_r} + 1)\right)^2 - 4\boldsymbol{A_r}(\epsilon - 1)\big(1 + \boldsymbol{A_r}(2 - \epsilon)\big)}}{2\big(1 + \boldsymbol{A_r}(2 - \epsilon)\big)}$$
$$Z_2^d = \frac{\frac{\epsilon - 1}{\epsilon}(\boldsymbol{A_r} + 1) + \sqrt{\left(\frac{\epsilon - 1}{\epsilon}(\boldsymbol{A_r} + 1)\right)^2 - 4\boldsymbol{A_r}(\epsilon - 1)\big(1 + \boldsymbol{A_r}(2 - \epsilon)\big)}}{2\big(1 + \boldsymbol{A_r}(2 - \epsilon)\big)}$$

and $\left(Z_1^d, Z_2^d\right) \in [0,1]^2$. We have, in that case,

$$N < 0 \iff Z \in [Z_1^d, Z_2^d]$$

Therefore, the symmetric configuration is unstable if, and only if,

$$\boldsymbol{A_r} < \frac{(2\epsilon + 1) - \sqrt{(2\epsilon + 1)^2 - \left(1 - 4\frac{2 - \epsilon}{\epsilon - 1}\right)}}{1 - 4\frac{2 - \epsilon}{\epsilon - 1}}$$
$$Z \in [Z_1^d, Z_2^d]$$

Proof 3: The full agglomeration equilibrium is a stable equilibrium if and only if the capital rate of return in the region where firms agglomerate is larger than in the other region. Let region 1 be the region where firms agglomerate. We then have $n_2 = 0$. Let us consider $r_1 - r_2$. From Equation (7), we have

$$r_1 - r_2 > 0 \quad \iff \quad p_1 q_1 - p_2 q_2 > 0$$
$$\iff \quad p_1 q_1 \left(1 - w_1^{\epsilon - 1}\left(\frac{1 - Z}{1 + Z}\left(\frac{\epsilon - 1}{\epsilon} + \frac{1}{2\epsilon}\right) + \frac{1 + Z}{1 - Z}\frac{1}{2\epsilon}\right)\right) > 0$$

$$\Leftrightarrow \quad \begin{cases} w_1 < \left(\dfrac{\epsilon^2}{2\epsilon - 1}\right)^{\frac{1}{2(\epsilon-1)}} \\ Z \in \ [Z_1^a, Z_2^a] \end{cases}$$

The full agglomeration is therefore a stable equilibrium if and only if

$$\boldsymbol{A_r} < \sigma\left(\frac{\epsilon^2}{2\epsilon - 1}\right)^{\frac{1}{2\sigma(\epsilon-1)}}$$
$$Z \in \ [Z_1^a, Z_2^a]$$

Proof 4:

$$A_d^* < A_a^* \quad \Leftrightarrow \quad \frac{(2\epsilon + 1) - \sqrt{(2\epsilon + 1)^2 - \left(1 - 4\dfrac{2-\epsilon}{\epsilon - 1}\right)}}{1 - 4\dfrac{2-\epsilon}{\epsilon - 1}} < \sigma\left(\frac{\epsilon^2}{2\epsilon - 1}\right)^{\frac{1}{2\sigma(\epsilon-1)}}$$

$$\Leftrightarrow \quad 1 - 4\frac{2-\epsilon}{\epsilon - 1} < 2\epsilon + 1$$

Note that $\forall \epsilon, 1 - 4\frac{2-\epsilon}{\epsilon-1} < 2\epsilon + 1$, thus $A_d^* < A_a^*$.

Proof 5: The symmetric configuration is a stable equilibrium if and only if a growth of the population in one region around the symmetric equilibrium leads to smaller utility in the largest region than in the smallest. Namely,

$$\frac{\boldsymbol{L_r}}{\boldsymbol{U_r}}\frac{d\boldsymbol{U_r}}{d\boldsymbol{L_r}}\Big|_{\lambda=\frac{1}{2}} < 0$$

From Equation (13), we have:

$$\frac{\boldsymbol{L_r}}{\boldsymbol{U_r}}\frac{d\boldsymbol{U_r}}{d\boldsymbol{L_r}}\Big|_{\lambda=\frac{1}{2}} < 0 \Leftrightarrow \frac{\left(\dfrac{(\epsilon-1)(\boldsymbol{A_r}+1)}{\epsilon} - Z\boldsymbol{A_r}\right)\left(\dfrac{\epsilon-1}{\epsilon}Z - 1\right)}{1 + \boldsymbol{A_r}\epsilon - Z\dfrac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1) + Z^2\boldsymbol{A_r}(1-\epsilon)} - \frac{1}{\epsilon} - \mu < 0$$

From Proof 2, we have:

$$D = 1 + \boldsymbol{A_r}\epsilon - Z\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1) + Z^2\boldsymbol{A_r}(1-\epsilon) > 0.$$

Thus,

$$\frac{\left(\frac{(\epsilon-1)(\boldsymbol{A_r}+1)}{\epsilon}-Z\boldsymbol{A_r}\right)\left(\frac{\epsilon-1}{\epsilon}Z-1\right)}{1+\boldsymbol{A_r}\epsilon-Z\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1)+Z^2\boldsymbol{A_r}(1-\epsilon)}<\frac{1}{\epsilon}$$
$$\Leftrightarrow Z\left(\frac{(\boldsymbol{A_r}+1)(\epsilon-1)}{\epsilon}+\boldsymbol{A_r}\right)<\frac{(\epsilon-1)(\boldsymbol{A_r}+1)+1+\boldsymbol{A_r}\epsilon}{\epsilon}$$
$$\Leftrightarrow Z\big((\boldsymbol{A_r}+1)(\epsilon-1)+\boldsymbol{A_r}\epsilon\big)<(\epsilon-1)(\boldsymbol{A_r}+1)+1+\boldsymbol{A_r}\epsilon$$

Note that $\forall\epsilon>1,\forall\boldsymbol{A_r}>0,\forall Z\in\ [0,1]$:

$$Z\big((\boldsymbol{A_r}+1)(\epsilon-1)+\boldsymbol{A_r}\epsilon\big)<(\epsilon-1)(\boldsymbol{A_r}+1)+1+\boldsymbol{A_r}\epsilon$$

Thus, as $\mu>0$, we have,

$$\frac{\left(\frac{(\epsilon-1)(\boldsymbol{A_r}+1)}{\epsilon}-Z\boldsymbol{A_r}\right)\left(\frac{\epsilon-1}{\epsilon}\ Z-1\right)}{1+\boldsymbol{A_r}\epsilon-Z\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1)+Z^2\boldsymbol{A_r}(1-\epsilon)}-\frac{1}{\epsilon}-\mu<0$$

Therefore, the symmetric configuration is always a stable equilibrium.

Proof 7: From Equation (15), we have:

$$\frac{\boldsymbol{L_r}}{\boldsymbol{r_r}}\frac{\delta\boldsymbol{r_r}}{\delta\boldsymbol{L_r}}=\frac{\boldsymbol{A_r}(\epsilon-1)(1-Z^2)}{1+\boldsymbol{A_r}\epsilon-Z\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1)+Z^2\boldsymbol{A_r}(1-\epsilon)}$$

From Proof 2, we have$D=1+\boldsymbol{A_r}\epsilon-Z\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1)+Z^2\boldsymbol{A_r}(1-\epsilon)>0$. Thus, as $\epsilon>1,\boldsymbol{A_r}>0$ and $Z>1$,

$$\frac{\boldsymbol{L_r}}{\boldsymbol{r_r}}\frac{\delta\boldsymbol{r_r}}{\delta\boldsymbol{L_r}}>0$$

From Equation (16), we have:

$$\frac{\boldsymbol{n_r}}{\boldsymbol{U_r}}\frac{\delta\boldsymbol{U_r}}{\delta\boldsymbol{n_r}}=(1-\delta^m-\delta^H)\frac{(\boldsymbol{A_r}+1)\left(\frac{\epsilon-1}{\epsilon}+Z\frac{1}{\epsilon-1}-Z^2\right)}{1+\boldsymbol{A_r}\epsilon-Z\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1)+Z^2\boldsymbol{A_r}(1-\epsilon)}$$

From Proof 2, we have $D=1+\boldsymbol{A_r}\epsilon-Z\frac{\epsilon-1}{\epsilon}(\boldsymbol{A_r}+1)+Z^2\boldsymbol{A_r}(1-\epsilon)>0$. Besides,

$$\forall\epsilon,\forall\boldsymbol{A_r},\forall Z,\qquad \boldsymbol{A_r}+1>0$$
$$\frac{\epsilon-1}{\epsilon}+Z\frac{1}{\epsilon-1}-Z^2>0$$

Therefore,

$$\frac{\boldsymbol{n_r}}{\boldsymbol{U_r}}\frac{\delta\boldsymbol{U_r}}{\delta\boldsymbol{n_r}}>0$$

Proof 8: The symmetric configuration is a stable equilibrium if and only if the eigenvalues of the Jacobian matrix $\boldsymbol{J}$ are negative, with $\boldsymbol{J}$ equal to:

$$\boldsymbol{J} = \begin{pmatrix} \frac{\boldsymbol{n_r}}{\boldsymbol{r_r}} \frac{\delta \boldsymbol{r_r}}{\delta \boldsymbol{n_r}} & \frac{\boldsymbol{n_r}}{\boldsymbol{r_r}} \frac{\delta \boldsymbol{r_r}}{\delta \boldsymbol{n_r}} \\ \frac{\boldsymbol{n_r}}{\boldsymbol{r_r}} \frac{\delta \boldsymbol{r_r}}{\delta \boldsymbol{n_r}} & \frac{\boldsymbol{n_r}}{\boldsymbol{r_r}} \frac{\delta \boldsymbol{r_r}}{\delta \boldsymbol{n_r}} \end{pmatrix}$$

From Proof 7, $\boldsymbol{J}$ is diagonalizable. Let V_1 and V_2 be the two eigenvalues of $\boldsymbol{J}$. From Equation (sets) (15) and (16), we have:

$$\begin{cases} V_1 < 0 \\ V_2 < 0 \end{cases} \Leftrightarrow \begin{aligned} &(-\epsilon(\epsilon \boldsymbol{A_r} + 1) + (1 + \boldsymbol{A_r})(\epsilon - 1)Z \\ &+ \epsilon \boldsymbol{A_r}(\epsilon - 1)Z^2)\Big(-(1+\mu)\big(1 + \epsilon(Z-1)\big)Z \\ &+ \boldsymbol{A_r}\Big(1 - (1+\mu)Z - \epsilon(Z-1)\big(\mu - 1 + Z(1+2\mu)\big) \\ &+ \epsilon^2\mu(Z^2 - 1)\Big)\Big) > 0 \end{aligned}$$

We note:

$$E = -\epsilon(\epsilon \boldsymbol{A_r} + 1) + (1 + \boldsymbol{A_r})(\epsilon + 1)Z + \epsilon \boldsymbol{A_r}(\epsilon - 1)Z^2$$

We thus have,

$$E < 0 \Leftrightarrow Z \in \left[Z_E^{(1)}, Z_E^{(2)}\right]$$

where,

$$Z_E^{(1)} = \frac{-(\boldsymbol{A_r} + 1)(\epsilon - 1) - \sqrt{(\epsilon - 1)((\epsilon - 1)(\boldsymbol{A_r} + 1)^2 + 4\epsilon^2 \boldsymbol{A_r}(\epsilon \boldsymbol{A_r} + 1))}}{2\epsilon \boldsymbol{A_r}(\epsilon - 1)}$$

$$Z_E^{(2)} = \frac{-(\boldsymbol{A_r} + 1)(\epsilon - 1) + \sqrt{(\epsilon - 1)((\epsilon - 1)(\boldsymbol{A_r} + 1)^2 + 4\epsilon^2 \boldsymbol{A_r}(\epsilon \boldsymbol{A_r} + 1))}}{2\epsilon \boldsymbol{A_r}(\epsilon - 1)}$$

However, $\forall \epsilon, \forall \boldsymbol{A_r}$,

$$\frac{-(\boldsymbol{A_r} + 1)(\epsilon - 1) - \sqrt{(\epsilon - 1)((\epsilon - 1)(\boldsymbol{A_r} + 1)^2 + 4\epsilon^2 \boldsymbol{A_r}(\epsilon \boldsymbol{A_r} + 1))}}{2\epsilon \boldsymbol{A_r}(\epsilon - 1)} < 0$$

$$\frac{-(\boldsymbol{A_r} + 1)(\epsilon - 1) + \sqrt{(\epsilon - 1)((\epsilon - 1)(\boldsymbol{A_r} + 1)^2 + 4\epsilon^2 \boldsymbol{A_r}(\epsilon \boldsymbol{A_r} + 1))}}{2\epsilon \boldsymbol{A_r}(\epsilon - 1)} > 1$$

Therefore, as $Z \in [0,1]$, we have $\forall \epsilon, \forall \boldsymbol{A_r}, E < 0$,

We note:

$$F = -(1+\mu)\big(1+\epsilon(Z-1)\big)Z + \boldsymbol{A_r}\Big(1-(1+\mu)Z-\epsilon(Z-1)\big(\mu-1+Z(1+2\mu)\big)+\epsilon^2\mu(Z^2-1)\Big)$$

Considering Δ, the discriminant, we have:

$$\Delta = C_d$$

where,

$$C_d = ((1+\mu)(\epsilon-1-\boldsymbol{A_r})+\epsilon\boldsymbol{A_r}(2+\mu))^2 + 4\epsilon\boldsymbol{A_r}(\epsilon-1)(1+\mu\epsilon)\big(\boldsymbol{A_r}(\mu\epsilon-1-2\mu)-(1+\mu)\big) > 0$$

Thus we have:

$$\Delta > 0 \Leftrightarrow C_d > 0$$

In this case, we have:

$$F > 0 \Leftrightarrow Z \in [Z_1^{*d}, Z_2^{*d}]$$

where,

$$Z_1^{*d} = \frac{(\boldsymbol{A_r}+1-\epsilon)(1+\mu)-\epsilon\boldsymbol{A_r}(2+\mu)-\sqrt{C_d}}{2\epsilon(\boldsymbol{A_r}(\mu\epsilon-2\mu-1)-(1+\mu))}$$

$$Z_2^{*d} = \frac{(\boldsymbol{A_r}+1-\epsilon)(1+\mu)-\epsilon\boldsymbol{A_r}(2+\mu)+\sqrt{C_d}}{2\epsilon(\boldsymbol{A_r}(\mu\epsilon-2\mu-1)-(1+\mu))}$$

and $\big(Z_1^{*d}, Z_2^{*d}\big) \in [0,1]^2$

Therefore, the symmetric configuration is unstable if and only if

$$C_d > 0$$
$$Z \in [Z_1^{*d}, Z_2^{*d}]$$

Proof 9: The full agglomeration equilibrium is a stable equilibrium for workers if and only if the utility in the region where workers agglomerate is larger than in the other region. Let region 1 be the region where population agglomerates. We then have $\lambda = 1$. From Equation (9), we have

$$\frac{U_1}{U_2} = \left(\left(\frac{P_1}{P_2}\right)\frac{\frac{2\epsilon-1}{2\epsilon}p_1\,n_1\,q_1+\frac{1}{2\epsilon}p_2\,n_2\,q_2}{\frac{2\epsilon-1}{2\epsilon}p_2\,n_2\,q_2+\frac{1}{2\epsilon}p_1\,n_1\,q_1}\right)^{1-\delta^a-\delta^H}\left(\frac{1-\lambda}{\lambda}\right)^{1-\delta^a}\left(\frac{1+\left(\frac{1-\lambda}{\kappa_2}\right)^{\alpha}}{1+\left(\frac{\lambda}{\kappa_1}\right)^{\alpha}}\right)^{\delta^a}$$

Note that

$$\left(\frac{P_1}{P_2}\right)\frac{\frac{2\epsilon-1}{2\epsilon}p_1\,n_1\,q_1+\frac{1}{2\epsilon}p_2\,n_2\,q_2}{\frac{2\epsilon-1}{2\epsilon}p_2\,n_2\,q_2+\frac{1}{2\epsilon}p_1\,n_1\,q_1} < \tau(2\epsilon-1)$$

thus,

$$\frac{U_1}{U_2} < (\tau(2\epsilon - 1))^{1-\delta^a-\delta^H}\left(\frac{1-\lambda}{\lambda}\right)^{1-\delta^a}\left(\frac{1+\left(\frac{1-\lambda}{\kappa_2}\right)^{\alpha}}{1+\left(\frac{\lambda}{\kappa_1}\right)^{\alpha}}\right)^{\delta^a}$$

Therefore,

$$\lim_{\lambda\to 1}\frac{U_1}{U_2} = 0$$

When $\lambda = 1, U_1 < U_2$. The full agglomeration of the population in one region is never a spatial equilibrium.