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Spatial Nonlinear Pricing with Per-Trip versus Per-Unit Transportation Costs*

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Abstract: In this paper, we introduce a nonlinear pricing model with transportation costs that do not vary with the quantity purchased. In contrast to the quantity discounts seen in a model with per-unit transportation charges, we find that the optimal nonlinear pricing schedule takes the form of either a two-part tariff or package pricing, which appears to be consistent with observed pricing structures. We compare efficiency measures between optimal nonlinear pricing and the single price monopoly result, finding that per trip transportation costs can lead to qualitatively different results for pricing structure and efficiency measures.

Keywords: monopoly, spatial pricing

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1. INTRODUCTION

Many spatial pricing rules have been examined for their effects on profits, firm location, and social welfare. The models nearly all assume that consumers face a separate shipping charge for each unit purchased, where the charge increases linearly in the distance to the seller. In these per-unit transportation cost models, the transportation cost enters the consumer's decision problem essentially as a simple increase in price. But, as Stahl (1981) points out in a critique of location theory, the per-unit shipping cost paradigm ignores the many cases for which the marginal shipping cost for additional units of a good are zero. Stahl (1987, p. 793) also argues that:

Most acts of consumption are preceded by a shopping trip, during which the relevant commodities are purchased and transferred to the consumer's home; alternatively, services are consumed at the location of delivery. Such shopping trips, especially if conducted by car, are characterized by great economies of scale in the quantity of any commodity or service picked up...

He therefore argues that economists should address a class of models with transportation costs that are nonconvex in quantity and paid once per trip regardless of the quantity purchased. We agree that the "shopping trip" phenomenon is common and that this class of model has received inadequate attention in the literature. Pizza delivery, vacations, and many home services such as cleaning and appliance repair are also characterized by per-trip transportation costs. It is important to understand how pricing structures and welfare measures are affected under these assumptions.

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The pricing theory literature has analyzed myriad choices for pricing structure. The simplest choice is the linear mill price, whereby every customer pays the same price for every unit that is purchased, and the customer is responsible for transporting the good to the place where it will be consumed (commonly called f.o.b. mill pricing). Various forms of price discrimination allow a firm to charge different prices to different groups of consumers based on some identifying characteristic of the consumer. In the case of spatial price discrimination, the firm can charge different prices to each consumer, based upon where the consumer “lives.” Another common spatial pricing method is to charge each customer the same price for each unit, where these prices include delivery. This type of pricing is commonly called “uniform delivered pricing.” In the shopping trip scenario described above by Stahl, however, only effective f.o.b. mill pricing by the retailer is a plausible option.

Two additional common pricing schemes are possible when the location of the buyer is unknown. A two-part tariff (Oi, 1971; Schmalensee, 1981) charges a fixed access fee plus a separate per-unit price, as is the case at a county fair (gate fee plus ride fees) or a warehouse membership club. A nonlinear pricing scheme (e.g., Spulber, 1981) allows the seller to charge a different price for each unit, which we often see done through quantity discounts. Either of these two schemes depends on knowing the general distribution of consumer locations, but do not depend on identifying an individual customer’s location.

In this paper we compare a monopolist’s linear and nonlinear pricing structures in the shopping-trip scenario and find that the optimal nonlinear pricing schedule can be simplified into a two-part tariff or a single package price. We derive and compare the pricing, market areas, quantity sold, profits, and consumer surplus under these two pricing policies, and show how these results are qualitatively different from what would be observed in a per-unit transportation cost scenario.

In the next section, we discuss the previous literature in spatial pricing models. In Section 3, we introduce the general pricing model with per-trip transportation costs. We then derive general results for linear and nonlinear pricing schemes in Section 4. We derive further results using specific functional forms in Section 5, followed by a numerical example in Section 6, and concluding remarks in Section 7.

2. LITERATURE REVIEW

In addition to the structure of transportation costs, spatial pricing models can also be differentiated by the industrial organization of market suppliers, e.g., whether it is assumed to be monopolistic or oligopolistic. In oligopolistic models sometimes the sellers’ locations are fixed, sometimes the focus is upon the choice of locations, and in some models the focus is upon the number of entrants into a market (e.g., Salop, 1979). The typical analysis of spatial pricing policies emphasizes the movement of large amounts of raw materials from a supplier to an industrial consumer. Additional units of goods purchased carry an additional nontrivial shipping charge, as might be true of purchasing an additional truckload or rail car of material. Since these goods are often delivered by the supplier, the supplier’s choice of spatial pricing policies abound. The most common are f.o.b. mill (e.g., Hotelling, 1929), uniform delivered, and spatial price discrimination. Additionally, the two-part tariff (e.g., Coyte and Lindsey, 1988 and Kanemoto, 2000) and nonlinear pricing (Spulber, 1981; Stole, 1985; Yin, 2004) have been explored in a spatial context. Greenhut and Greenhut (1977) studied nonlinear delivered pricing schedules as a form of spatial price discrimination. They found that nonlinear delivered pricing naturally arises as a more

profitable pricing strategy than mill pricing under most functional forms for demand. Pires and Sarkar (2000) examined nonlinear delivered pricing under duopoly in which each firm has different costs. Spulber (1981) examined nonlinear mill pricing in a spatial context and found that quantity discounts improve profits for a monopolist with per-unit transportation costs.

All of the previously mentioned studies examine per-unit transportation costs. The distinction between per-trip and per-unit costs is irrelevant when consumers inelastically purchase exactly one unit (e.g., Hotelling, 1929; Eaton and Lipsey, 1975; Villas-Boas and Schmidt-Mohr, 1999; Jorge and Pires, 2008) or have a so-called “rectangular” demand function¹ (Lerner and Singer 1937; Beckmann, 1976). But only a handful of studies examine per-trip transportation costs.

Burkey (2011) used a linear demand function to derive optimal pricing policies under several pricing policies. He finds that in cases of per-trip transportation costs, the best policy involves subsidizing transportation costs rather than discriminating on prices. Bacon (1971) and Eaton and Lipsey (1979, 1982) studied the optimal location of firms, focusing on the agglomeration forces for given prices. Stahl (1982) analyzed a model where customers shop for bundles of various goods and want to minimize these per-trip costs by shopping at locations offering a variety of goods. Similarly, Bliss (1988) assumed a per-trip transportation cost when discussing the function of the retail store. Anderson, de Palma, and Thisse (1992) briefly discussed the implications of per-trip transportation costs on a duopolistic location model. This type of location model is discussed further by Rath and Zhao (2001) in a two-stage game in which locations are chosen in the first stage and prices are chosen in the second stage.

Armstrong and Vickers (2001) used a per-trip transportation cost model to focus on how competition affects the ability of firms to use price discrimination in an oligopolistic context. These authors add much insight into how structural and informational constraints affect welfare distributions; but they do not focus on how per-trip transportation costs change firm and consumer behavior and welfare as we do in this paper. In what follows, the focus is on the predicted changes in behavior that the shopping trip scenario implies for both firms and consumers, and the resulting welfare implications for policy makers.

3. PER-TRIP TRANSPORTATION COST MODEL

We consider the problem of a monopolist selecting an optimal pricing policy when its customers are spatially dispersed; we do so following the general modeling framework and notation of Spulber (1981). The monopolist sets a marginal pricing schedule for its customers that depends on the quantity purchased. The monopolist is located at point 0 on real linear space, with customers a distance r from the monopolist ($r \in [0, \infty)$). Customers are located continuously on the linear space according to a cumulative distribution $G(r)$ where $G(0)=0$ and $g(r) \equiv dG(r)/dr$. The consumers are identical in all respects except for location. The consumer faces exogenous transportation costs, $C(r)$, that increase in r but not in quantity. The consumer maximizes a utility function of the good of interest, x , and a unit-priced numeraire, y : $U(x,y)=u(x)+y$, where u is twice differentiable, such that $\partial u/\partial x > 0$, $\partial^2 u/\partial x^2 < 0$. Assuming income effects are negligible, we can write the consumer's objective function as:

¹ Such a demand function has demand for one unit unless the cost exceeds some upper limit.

$$(1) \quad \max_x [u(x) - P(x) - C(r)]$$

where $P(x)$ represents the *total* purchase price of x units. Faced with fixed transportation costs, the consumer chooses to purchase from the monopolist only if his consumer's surplus² (CS) is at least as large as the per-trip transportation cost. For simplicity, we assume that the per-trip travel cost function is linear in r , so that the cost of the trip faced by a consumer located at r is Cr , with $C \geq 0$. At all times in this paper, we assume that prices are f.o.b. mill, i.e. transportation costs are borne by the consumer.

We can state the consumer's decision problem as:

Purchase x^* s.t. $u'(x) = P'(x)$ if $CS = u(x) - P(x) \geq Cr$, 0 otherwise.

Per-trip transportation costs have no effect on the quantity of x purchased by a consumer, but have cause some consumers to exit the market if costs run too high. The transportation cost will determine, for a given price, the point on the real line at which consumers exit:

$$(2) \quad r^o = \frac{u(x^*) - P(x^*)}{C}$$

In this paper we consider two specific cases for $P(x)$. First, we will consider the case where the same price is charged for each unit—we call this the linear price. We do not use the more common term “mill price,” since the price paid at the mill could obviously be linear, nonlinear, a two-part tariff, or other pricing policy. Then, we will consider the case where a nonlinear pricing function can be used.

4. RESULTS WITH GENERAL FORMS

Assuming a constant marginal cost of production ($k > 0$) for the monopolist, the monopolist will choose the profit-maximizing price (or nonlinear pricing schedule). In this section, we derive the optimal pricing strategies for both linear and nonlinear pricing schemes. In Proposition 1 we consider the linear price, followed by the nonlinear case in Proposition 2.

Proposition 1: *Using the linear pricing function $P(x) = px$, the profit-maximizing monopoly price satisfies:*

$$(3) \quad p = k + \frac{G[r^o(P)]C}{x^*(p)[G[r^o(P)] - \frac{\partial x^*}{\partial p} G[r^o(P)]C / x^*(p)^2]}$$

Proof: The monopolist chooses p to maximize the profit function:

$$(4) \quad \max_p (p - k) \int_0^{r^o(p)} x^*(p) g(r) dr = \max_p (p - k) G[r^o(P)] x^*(p)$$

² Defined as the difference between the maximum willingness to pay for a given quantity of a product and the price paid.

$$(5) \quad \frac{\partial}{\partial p} = x^*(p)G[r^o(P)] + (p-k)\frac{\partial G}{\partial r^o}\frac{\partial r^o}{\partial p}x^*(p) + (p-k)\frac{\partial x^*}{\partial p}G[r^o(P)] = 0$$

Rearranging (5) and using Equation (2) gives (3). The last term in (3) is positive, because all arguments in the function are positive with the exception of $\partial x^*/\partial p$, which is negative but preceded by a (-) sign. $\square$

Proposition 1 states that the price will reflect a markup above marginal cost (k) that is increasing in the transportation rate (C) and in the cumulative number of consumers within the market who connect, but it is decreasing with the number of consumers as their density around the exit point increases since there is an incentive to lower the price to add these consumers to the market. In other words, as the number of consumers who are buying rises, the monopolist can afford to raise the price; but as the number living near the exit point increases, the monopolist can gain many more customers by lowering the price ever so slightly. This relationship can be rewritten as a familiar profit-maximization condition that relates marginal cost to price as well as to the price elasticity of demand:

$$(6) \quad k = p \left(1 + \frac{1}{\left[\frac{p}{x^*(p)G(r^o(p))} \right] \left[\frac{\partial x^*}{\partial p} G[r^o(P)] + \frac{\partial G}{\partial r^o} \frac{\partial r^o}{\partial p} x^*(p) \right]} \right) = p \left(1 + \left(\frac{1}{\frac{dx}{dp} \frac{p}{x}} \right) \right) = p \left(1 + \frac{1}{\varepsilon_p} \right)$$

For linear monopoly pricing, profit maximization entails finding a price such that the marginal cost has a relationship with the price and ε_p shown in Equation (6). The denominator of (6) is simply the marginal effect on sales from changes in price. The first term in brackets in the denominator is the “inframarginal” effect of price, reflecting the decrease in the quantity purchased by those who purchase. The second bracketed term in the denominator is the “extramarginal” effect, reflecting the change in sales caused by changes in the market length.

Next we consider a nonlinear pricing function $P(x)$ and prove the following proposition.

Proposition 2: *The optimal nonlinear price for the monopolist satisfies the equation:*

$$(7) \quad P(x^*) = kx^* + \frac{G[r^o(P)]C}{g[r^o(P)]}$$

where x^* is the optimal quantity purchased by all consumers who connect to the market, and $P(x)$ is the f.o.b. package price charged to each consumer for x^* units.

Proof: With a per-trip travel cost and identical demand functions at each point along the real line, all consumers up to the exit point will purchase the same amount of x (x^* s.t. $u'(x) = P'(x)$), and all consumers whose travel cost is greater than their consumer surplus will purchase no x at all. Given this, the monopolist’s problem is to choose a nonlinear pricing schedule that optimizes both the quantity, x^* , for all who purchase and the size of the market area through r^o in Equation (2). This problem is intuitively similar to the linear-pricing problem above. For any consumer who purchases, given a total price $P(x)$, consumers will maximize:

$$(8) \quad \max_x [u(x) - Cr - P(x)], \text{ where}$$

$$(9) \quad u'(x) = P'(x)$$

Let $p(x) \equiv dP(x)/dx$. The monopolist will maximize

$$(10) \quad \max_{P(x)} \int_0^{r^o} \int_0^{x^*} (p(x) - k)g(r)dxdr$$

Solving for a unique marginal pricing function degenerates into the per-trip transportation cost problem, because the marginal price (and hence the quantity purchased) will be constant for each consumer who connects. In this case, the problem can be restated in a much simpler framework. Given that all consumers who connect will purchase the same quantity, x^* , the monopolist can first determine the profit-maximizing quantity, and then determine the profit-maximizing total price, $P(x^*)$, based on the amount of disconnection that will occur by changing this price at the margin.

It is well known that under optimal nonlinear pricing, the marginal price at the equilibrium quantity must be set equal to marginal cost. The monopolist will want to choose x^* for each consumer who purchases where

$$(11) \quad u'(x^*) = P'(x^*) = k$$

Note that Equation (11) is consistent with condition (7) for the total price $P(x^*)$, since pricing at marginal cost will maximize the consumer surplus and the number of consumers connecting for any total price $P(x^*)$. Given this x^* , the monopolist needs to choose the total price to maximize profits. The choice of $P(x^*)$ will affect the number of consumers who connect to the market. From (2), we have $r^o = \frac{u(x^*) - P(x^*)}{C}$. Thus, the monopolist's maximization problem is

$$(12) \quad \max_P (P - kx^*) \int_0^{r^o(P)} g(r)dr = \max_P (P - kx^*) G[r^o(P)]$$

Differentiating (12) w.r.t. P gives the first-order condition:

$$(13) \quad \frac{\partial}{\partial P} \Rightarrow \int_0^{r^o(P)} g(r)dr - (P - kx^*)g(r^o)\frac{1}{C} = 0$$

$$\Rightarrow G(r^o) - (P - kx^*)g(r^o)\frac{1}{C} = 0$$

which implicitly defines the profit-maximizing P as:

$$(14) \quad P(x^*) = kx^* + \frac{G[r^o(P)]C}{g[r^o(P)]} \quad \square$$

Equation (14) is similar to (3) in that the profit-maximizing price increases as the transportation cost per unit distance increases and as the number of customers within the market area increases. However, as the density of consumers around the exit point increases, the monopolist feels pressure to lower the price of x in order to keep customers within the extensive margin. This total price could be implemented with any nonlinear pricing schedule that induces consumers to purchase x^* as described above. One such function would be to charge an admission

fee or other fixed charge equal to $(G[r^o(P)]C/g[r^o(P)])$, and then charge k per unit purchased.

As discussed above, this admission fee critically depends on the consumer surplus and density of consumers near the disconnection point. By charging a fixed fee and marginal cost unit pricing, our result is essentially a two-part tariff.³ Alternatively, the monopolist could simply charge a package price for x^* units. No matter how it is accomplished, a marginal pricing schedule must charge k (marginal cost) for the last unit purchased. This is a qualitatively very different result from how nonlinear pricing is structured when transportation costs are per-unit. Spulber (1981) found that the optimal nonlinear pricing schedule with per-unit transportation costs involves quantity discounts, and that consumers purchase smaller amounts as they live farther from the mill. In the business to business model, where per-unit transportation costs are more prevalent, quantity discounts are the norm. In a study of business pricing arrangements Munson and Rosenblatt (1998) found that “Eighty-three percent of the buyers in the field study receive quantity discounts for *most* of the items that they purchase...” (p.365). A survey of *consumer* shopping would likely not find that the same can be said, where transportation costs are normally per trip. Rather, most consumer products are sold in various package sizes, which may or may not involve a discount for purchasing a larger quantity.⁴

Now we compare the results under the two pricing arrangements. Keep in mind that because per-trip transportation costs do not affect the quantity purchased, and because the nonlinear pricing scheme is essentially a two-part tariff, that all consumers who connect to the market will purchase the same amount. Let x_l^* and x_{nl}^* denote the quantity purchased by each consumer under the linear and nonlinear pricing arrangements, respectively.

Proposition 3: *For each consumer that connects, $x_{nl}^* \geq x_l^*$.*

Proof: Suppose that $x_{nl}^* < x_l^*$. Since $u'(x)$ is a decreasing function we have $u'(x_{nl}^*) > u'(x_l^*)$. From Proposition 2, the nonlinear price sets the quantity purchased s.t. $u'(x_{nl}^*) = k$ for the last unit purchased. Customers under the linear price will purchase an amount s.t. $u'(x_l^*) = p$. Hence, $k = u'(x_{nl}^*) > u'(x_l^*) = p$ which is a contradiction to Proposition 1, which states that the profit-maximizing linear price $p > k$. $\square$

Proposition 3 illustrates the well-known benefit of nonlinear pricing, which is to induce consumers to purchase a larger quantity without having to reduce the price of all units sold. Of course, this result holds for per-trip as well as per-unit travel cost situations. This is why nonlinear pricing will almost always enhance profits.⁵

It is difficult to take the analysis much further in a general model. Most other results such as consumer surplus and profits cannot be directly compared without many restrictions on the

³ See Oi (1971) or Ng and Weisser (1974) for detailed analyses of these models. This result is similar to that found by Armstrong and Vickers (2001) in an oligopolistic setting.

⁴ It is well-known that larger quantity packages often carry a higher per-unit cost. Gerstner (1985) finds modest evidence of lower unit prices for larger packages, which may have to do with packaging costs.

⁵ Greenhut and Greenhut (1977) show that for per-unit transportation costs a narrow class of utility functions will yield no improvement with nonlinear pricing.

functional forms. In the following section we derive additional results using some simple explicit forms for utility and distribution of consumer locations as used in Spulber (1981).

5. ANALYSIS WITH EXPLICIT FUNCTIONAL FORMS

In order to derive more results, and for comparison of results with models involving per-unit transportation costs, we now assume two common explicit functional forms for utility and the distribution of consumers:⁶

$$(15) \quad u(x) = \alpha x - (1/2)\beta x^2, \alpha > 0, \beta > 0$$

$$(16) \quad G(r) = ar^b, a > 0, b > 0$$

With these specific functional forms, the optimal linear price can be solved from:

$$(17) \quad \max_p (p - k)G[r^o(P)]x^*(p)$$

Using Equation (15), the demand function $x_l^*(p)$ satisfies $\max_x [u(x) - px]$, so that

$$(18) \quad x_l^*(p) = \frac{\alpha - p}{\beta}$$

Combining (15), (18), and (2) we can find the market length under linear pricing:

$$(19) \quad r_l^o = \frac{(\alpha - p)^2}{2C\beta}$$

By using Proposition 1 and combining (16), (18), and (19), we obtain the linear monopoly price and the quantity demanded by each consumer who purchases:

$$(20) \quad p_l = \frac{k(2b+1) + \alpha}{2(b+1)}$$

$$(21) \quad x_l^* = \frac{\alpha - k}{\beta} \frac{2b+1}{2b+2}$$

On the other hand, in the nonlinear pricing case we derive the quantity purchased by connecting consumers using the fact $u'(x_{nl}^*) = k$ [see Equation (11)], which together with (15) yields

$$(22) \quad x_{nl}^* = \frac{\alpha - k}{\beta}$$

Obviously (22) > (21), which for these explicit functional forms demonstrates a special case of Proposition 3.

To derive the total nonlinear price paid for these explicit functional forms, we use (16) in Proposition 2 and obtain

⁶ Following Spulber (1981).

$$(23) \quad P(x^*) = kx^* + \frac{r^0 C}{b}$$

Combining (2), (22) and (23), implies:

$$(24) \quad P_{nl} = \frac{(\alpha - k)(\alpha + 2bk + k)}{2\beta(1 + b)}$$

Proposition 4: *The average price charged per unit is the same for the linear price and the nonlinear pricing scheme.*

Proof: For the explicit solutions we have

$$p_l = \frac{k(2b+1) + \alpha}{2(b+1)} \text{ and } P_{nl} = \frac{(\alpha - k)(\alpha + 2bk + k)}{2\beta(1 + b)}$$

$$\text{Average nonlinear price} = \frac{P_{nl}}{x_{nl}^*} = \frac{\frac{(\alpha - k)(\alpha + 2bk + k)}{\beta 2(b+1)}}{\frac{\alpha - k}{\beta}} = \frac{k(2b+1) + \alpha}{2(b+1)} = p_l. \quad \square$$

While a nonlinear pricing scheme would charge higher prices for earlier units and k for the last unit purchased (or an equivalent package price), on average they are equal.⁷ However, as shown in Proposition 3, the monopolist induces each connecting consumer to purchase more with the nonlinear pricing scheme.

Proposition 5: *The market area served is larger under the linear pricing scheme, i.e., $r_l^o > r_{nl}^o$.*

Proof: From (19) and (20) we see that

$$(25) \quad r_l^o = \frac{(\alpha - k)^2 (2b + 1)^2}{8\beta C (b + 1)^2}$$

On the other hand, from (23) we have

$$(26) \quad r_{nl}^o = \frac{(P(x_{nl}^*) - kx_{nl}^*)b}{C}$$

which together with (22) and (24) yield

$$(27) \quad r_{nl}^o = \frac{(\alpha - k)^2 b}{2\beta C (b + 1)}$$

By canceling common terms in (25) and (27), it can be easily shown that $r_l^o > r_{nl}^o$. Thus the market area is larger for the linear pricing case. $\square$

⁷ This result is not true for other demand structures. For example, with a linear demand function the average price is higher under nonlinear pricing.

Because the transportation costs are per trip, Proposition 5 also shows that because the market area is larger under linear pricing, the number of consumers and the total transportation costs incurred will also be larger.

Proposition 6: *The profits are greater for a nonlinear pricing scheme than for a linear pricing scheme.*

In order to derive the profits in the linear case, we substitute (16), (19), (20), and (21) into the maximand (4). Simplifying gives

$$(28) \quad \pi_l = \frac{a(\alpha - k)^{2b+2} (2b+1)^{2b+1}}{4\beta^{b+1} 8^b (b+1)^{2b+2} C^b}$$

A similar procedure provides the profit for the nonlinear pricing case:

$$(29) \quad \pi_{nl} = \frac{a(\alpha - k)^{2b+2} b^b}{2^{b+1} \beta^{b+1} (b+1)^{b+1} C^b}$$

Due to its length and complexity, the proof of Proposition 6 is contained in the appendix to this paper, though it is intuitive that the more flexible nonlinear pricing would increase profits. Next, we derive the impact that nonlinear pricing has on the consumers.

Proposition 7: *The consumer surplus is greater for a linear pricing scheme than for a nonlinear pricing scheme.*

Using our assumed functional forms makes the derivation of consumer surplus straightforward because of the “numeraire good” assumption. In this case, CS is simply the maximand in (1). Keeping in mind that all consumers who connect to the market purchase the same amount at the same price, we can quantify consumer’s surplus by first finding $u(x) - P(x^*)$ for one consumer, and multiplying by the quantity of consumers who connect. Then, we can net out the total transportation costs. $P(x^*)$ will be either $p_l x^*$ for the linear pricing case, or the f.o.b. package price paid under the nonlinear scheme. Following this procedure gives

$$(30) \quad CS_l = \frac{a(2b+1)^{2b+2} (\alpha - k)^{2b+2}}{8^{b+1} \beta^{b+1} (b+1)^{2b+3} C^b}$$

$$(31) \quad CS_{nl} = \frac{ab^{b+1} (\alpha - k)^{2b+2}}{2^{b+1} \beta^{b+1} (1+b)^{b+2} C^b}$$

Proof of Proposition 7:

We will prove that the ratio of linear/nonlinear consumer surplus is greater than 1.

$$\frac{CS_l}{CS_{nl}} = \frac{(2b+1)^{2b+2} b^{b+1} (b+1)^{b+2}}{8^{b+1} (b+1)^{2b+3} b^{b+1}} \quad \text{which simplifies to:}$$

$$\begin{aligned} \frac{(2b+1)^{2b+2}}{4^{b+1}(b+1)^{b+1}b^{b+1}} &= \frac{(2b+1)^{b+1}(2b+1)^{b+1}}{4^{b+1}(b+1)^{b+1}b^{b+1}} = \left(\frac{(2b+1)(2b+1)}{4(b+1)b} \right)^{b+1} \\ &= \left(\frac{(4b^2+4b+1)}{4b^2+4b} \right)^{b+1} > 1 \end{aligned}$$

since $b > 0$. $\square$

Proposition 8: *Social surplus, defined as the sum of the profits and consumer surplus, are greater for a nonlinear pricing scheme than for a linear pricing scheme.*

Graphical proof: Total Social Welfare: Comparison of Profit + CS

Taking the ratio $\frac{\pi_{nl} + CS_{nl}}{\pi_l + CS_l}$ and simplifying, we get $\frac{4^{1+b}(1+b)^{1+b}b^b}{(4b+3)(2b+1)^{2b}}$, which is always greater than one. Thus, total social welfare under the nonlinear case is always higher than under a linear case. In Figure 1, we can see this ratio as a function of b . The nonlinear and linear surpluses converge as b become larger (i.e., one- or two-dimensional uniform distributions), but there is a significant difference between them for small values of b (e.g., high concentrations of consumers in the city core with low density in the suburbs). A formal proof is contained in the appendix.

Proposition 9: *The total number of units sold to the market is greater under a nonlinear pricing scheme than under the linear pricing scheme.*

Proposition 9 follows directly from the proofs of Propositions 4 and 6. Proposition 4 demonstrates that the average price per unit is the same under both schemes, but Proposition 6 shows that the profits are higher. Therefore, the quantity sold must be larger in the nonlinear pricing scheme. In fact, the ratio of total sales between the two pricing schemes is identical to the ratio of profits seen in Proposition 6.

Figure 1: Ratio of Nonlinear to Linear Surplus Converges to 1 as b Increases.

6. NUMERICAL EXAMPLE AND VISUALIZATION

Let us assume that consumers are distributed uniformly with density one ($b=1$ and $a=1$) for $r \in [0, \infty)$, and that each consumer has the inverse demand line $p'(x) = 10 - 2x$ ($\alpha=10$ and $\beta=2$). Lastly, let us assume that the marginal cost of production is $k=2$, and that it costs consumers $C=1$ per measure of distance (e.g., mile) to make the shopping trip to the monopolist's location. In this case, the optimal linear price is \$4, and each consumer who purchases will buy 3 units. The market length is $r^o=9$, because the consumer surplus for each consumer is 9. In Figure 2, box A is the cost of production for the monopolist, box B is the profit (\$54) at the optimal price, and area C illustrates the Consumer Surplus at the optimal price. If the monopolist were to raise its price to \$5, we can see the two impacts on profit (box D). The inframarginal effect is seen along the x -axis as existing customers decrease the quantity purchased from 3 to 2.5 units. The extramarginal effect can be seen along the r -axis as the market length decreases to 6.25 from 9, because the consumer surplus at this price is smaller (area E).

But as we have shown, the monopolist can increase profits by charging a two-part tariff or package price, as shown in Figure 3. By charging a total price of 16 for 4 units (or access fee of 8 (area B in Fig. 3) and per-unit charge of 2 (area A)), the profits for the monopolist are area B in Figure 3. In this case the market length is $r^o=8$, which is equal to the consumer surplus (area C).

Figure 2: Optimal Linear Price =4 (with Profit B), Suboptimal Price =5 (with Profit D)

Figure 3: Nonlinear price: Fixed Fee=8, Per-Unit Price=2

7. CONCLUSION

We have described the profit-maximizing behavior of a monopolist when transportation costs are per trip, as in most consumer markets. In Proposition 2 we found that the optimal nonlinear price in such a market could be charged as a two-part tariff, i.e. a fixed access fee plus marginal cost pricing for all units purchased. Still, since all similar connecting consumers will purchase the same quantity without regard to the distance they must travel, this pricing policy can easily be implemented by selling a standard package size. This type of pricing and consumer behavior conforms to what we commonly see in consumer markets, in contrast to the quantity discounts and lower quantities purchased by consumers on the market periphery predicted by per-unit models. In Proposition 4, we saw that for one class of utility and consumer density functions, the average price per unit purchased will be the same. Though the average price is the same, by choosing the nonlinear pricing strategy the sales will increase per customer, but the number of customers will decrease. Overall, however, this results in higher sales and profits for the seller.

We derived some clear implications for policy when transportation costs are per trip, whereas the efficiency results for per-unit models are not as clear. On aggregate, consumer surplus is always lower under nonlinear pricing, while total surplus is higher.

This paper advances our knowledge of per-trip models in the specific case of homogeneous consumers and a monopoly producer. Future research can expand our knowledge by analyzing additional pricing rules, oligopoly models, and multiplant monopolies. Additionally, because consumers will economize on shopping trips, a problem akin to the optimal stocking problem in inventory control can be analyzed in a more dynamic framework.

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Appendix

Proof of Proposition 6:

In order to show that the profits in the nonlinear case are higher, we start by taking the ratio of Equation 29 and Equation 28, and cancelling common terms and simplifying. We then need to show that for all $b > 0$, the following is true: $2^{2b+1}(b+1)^{b+1}b^b > (2b+1)^{2b+1}$,

hence,

$$2^{b+1}(b+1)^{b+1}2^b b^b > (2b+1)^{b+1}(2b+1)^b.$$

Thus

$$\left(\frac{2b+2}{2b+1}\right)^{b+1} > \left(\frac{2b+1}{2b}\right)^b$$

Therefore, in order to prove that the profit under nonlinear pricing is greater than under linear pricing we have to prove the following inequality

$$\left(1 + \frac{1}{2b+1}\right)^{b+1} > \left(1 + \frac{1}{2b}\right)^b \text{ for all } b > 0.$$

Proposition A. Let $f(b) = \left(1 + \frac{1}{2b}\right)^b$ and $g(b) = \left(1 + \frac{1}{2b+1}\right)^{b+1}$. Then the following holds:

- i) $\lim_{b \rightarrow 0} f(b) = 1$, and $g(0) = 2$,
- ii) $\lim_{b \rightarrow \infty} f(b) = \sqrt{e}$, and $\lim_{b \rightarrow \infty} g(b) = \sqrt{e}$,
- iii) $f'(b) > 0$, and $g'(b) < 0$ for $b > 0$.

Furthermore,

$$f(b) < g(b) \text{ on } (0, \infty).$$

Proof. Using L'Hopital's rule we have

$$\lim_{b \rightarrow 0} b \ln \left(1 + \frac{1}{2b}\right) = \lim_{b \rightarrow 0} \frac{\ln(1 + \frac{1}{2b})}{\frac{1}{b}} = \lim_{b \rightarrow 0} \frac{2b}{4b+2} = 0.$$

Hence, $\lim_{b \rightarrow 0} f(b) = e^0 = 1$. The fact that $g(0) = 2$, follows directly, thus part i is proven. Similarly Using L'Hopital's rule again we obtain

$$\lim_{b \rightarrow \infty} b \ln \left(1 + \frac{1}{2b}\right) = \lim_{b \rightarrow \infty} \frac{\ln(1 + \frac{1}{2b})}{\frac{1}{b}} = \lim_{b \rightarrow \infty} \frac{2b}{4b+2} = \frac{1}{2},$$

thus $\lim_{b \rightarrow \infty} f(b) = e^{(1/2)} = \sqrt{e}$. Similarly, we have that $\lim_{b \rightarrow \infty} g(b) = \sqrt{e}$, which proves ii.

Next we prove that $f(b)$ is a strictly increasing function on $(0, \infty)$ and that $g(b)$ is a strictly decreasing function on $(0, \infty)$. From the definition of f and the fact that $f(b) = e^{b \ln(1+(1/2b))}$ we have

$$f'(b) = \left(1 + \frac{1}{2b}\right)^b \left[\ln\left(1 + \frac{1}{2b}\right) - \frac{1}{2b+1} \right].$$

In order to prove that $f'(b) > 0$, we let

$$h(b) = \ln\left(1 + \frac{1}{2b}\right) - \frac{1}{2b+1}$$

and show that $h(b)$ is a strictly positive function on $(0, \infty)$. From the definition of h we see that $\lim_{b \rightarrow \infty} h(b) = 0$, and we can easily find that $h'(b) = -\frac{1}{b(2b+1)^2} < 0$ and $h''(b) > 0$. Therefore h is a strictly decreasing, concave upward function on $(0, \infty)$, asymptotically approaching zero. Thus, $h(b) > 0$, which in turn proves that $f'(b) > 0$ on $(0, \infty)$.

On the other hand, in order to prove that g is strictly decreasing we have

$$\begin{aligned} g'(b) &= \left(1 + \frac{1}{2b+1}\right)^{b+1} \left[\ln\left(1 + \frac{1}{2b+1}\right) + (b+1) \frac{-2(2b+1)}{(2b+2)(2b+1)^2} \right] \\ &= \left(1 + \frac{1}{2b+1}\right)^{b+1} \left[\ln\left(1 + \frac{1}{2b+1}\right) - \frac{1}{2b+1} \right]. \end{aligned}$$

Let $k(b) = \ln\left(1 + \frac{1}{2b+1}\right) - \frac{1}{2b+1}$. We see that $k(0) = \ln 2 - 1 < 0$, and $\lim_{b \rightarrow \infty} k(b) = 0$. Also, since

$k'(b) = \frac{1}{(b+1)(2b+1)^2} > 0$, and $k''(b) < 0$, we conclude that k is a strictly increasing and concave downward asymptotically approaching zero from below, thus $k(b) < 0$, which in turn proves that $g'(b) < 0$ on $(0, \infty)$.

Thus we have shown that f is a strictly increasing asymptotically approaching $\sqrt{e}$ from below and g is strictly decreasing asymptotically approaching $\sqrt{e}$ from above. That proves that $f(b) < g(b)$ on $(0, \infty)$ which completes the proof of Proposition A. $\square$

Proof of Proposition 8

We need to show that $4^{1+b}(1+b)^{1+b}b^b > (4b+3)(2b+1)^{2b}$ for $b>0$.

Proof:

From the above we see that

$$(4b+4)(4b^2+4b)^b > (4b+3)(4b^2+4b+1)^b,$$

hence

$$\frac{4b+4}{4b+3} > \left(\frac{4b^2+4b+1}{4b^2+4b} \right)^b$$

Therefore, in order to show that the total welfare under nonlinear case is always greater than under linear case we need to prove the following inequality

$$1 + \frac{1}{4b+3} > \left(1 + \frac{1}{4b^2+4b} \right)^b.$$

Since $b>0$, by taking the natural logarithm of both sides and applying the results of Proposition B below the proof of Proposition 6 follows directly.

Proposition B.

Let $\phi(b) = \ln(1 + \frac{1}{4b+3}) - b \ln(1 + \frac{1}{4b^2+4b})$. Then the following holds:

i) $\lim_{b \rightarrow 0} \phi(b) > 0$,

ii) $\lim_{b \rightarrow \infty} \phi(b) = 0$,

iii) $\phi''(b) > 0$, for all $b>0$.

Moreover $\phi(b) > 0$, for all $b>0$.

Proof. Using L'Hopital's rule we see that

$$\lim_{b \rightarrow 0} b \ln(1 + \frac{1}{4b^2+4b}) = \lim_{b \rightarrow 0} \frac{\ln(1 + \frac{1}{4b^2+4b})}{\frac{1}{b}} = \lim_{b \rightarrow 0} \frac{\frac{4b^2+4b}{4b^2+4b+1} \left(-\frac{8b+4}{(4b^2+4b)^2} \right)}{-\frac{1}{b^2}} = \lim_{b \rightarrow 0} \frac{b}{(2b+1)(b+1)} = 0.$$

Thus,

$$\lim_{b \rightarrow 0} \phi(b) = \ln(\frac{4}{3}) > 0,$$

which proves i). Similarly using L'Hopital's rule again we obtain

$$\lim_{b \rightarrow \infty} b \ln\left(1 + \frac{1}{4b^2 + 4b}\right) = \lim_{b \rightarrow \infty} \frac{b}{(2b+1)(2b+1)} = 0.$$

Thus,

$$\lim_{b \rightarrow \infty} \phi(b) = 0,$$

which proves ii). Next we show that

$$\begin{aligned} \phi'(b) &= \frac{4b+3}{4b+4} \left(\frac{-4}{(4b+3)^2} \right) - \left[\ln\left(1 + \frac{1}{4b^2+4b}\right) + b \frac{4b^2+4b}{4b^2+4b+1} \left(\frac{-8b-4}{(4b^2+4b)^2} \right) \right] \\ &= -\frac{1}{(b+1)(4b+3)} + \frac{1}{(2b+1)(b+1)} - \ln\left(1 + \frac{1}{4b^2+4b}\right) \\ &= \frac{2}{(4b+3)(2b+1)} - \ln\left(1 + \frac{1}{4b^2+4b}\right). \end{aligned}$$

Furthermore, we see that

$$\phi''(b) = -\frac{32b+20}{(2b+1)^2(4b+3)^2} + \frac{2b+1}{(2b+1)^2 b(b+1)} = \frac{1}{(2b+1)^2} \left(\frac{12b^2+4b+9}{b(b+1)(4b+3)^2} \right) > 0,$$

for all b ($b > 0$ by assumption), which proves part iii).

From i – iii we see that ϕ is a concave upward function asymptotically approaching 0 from above (with $\phi(0)$ positive); therefore, ϕ has to be positive for all $b > 0$. That completes the proof of Proposition B. $\square$