MARKET SHARE, DISTANCE, AND POTENTIAL*

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Summary

Suppose that in every location the ratio of sales by two firms offering heterogeneous products depends only on the difference of economic distance from the two plants, suitably defined. Then the market share is a simple function of distances. Market areas, defined as those subregions where market share of some firm dominates that of any other firm, are identical with conventional market areas for sellers of a homogeneous product. Next curves of equal market share--iso-share lines--are introduced, discussed, and calculated for special cases. It is shown that under certain restrictions on the function which specifies the distance effect and on the measure of economic distance, the present approach is consistent with demand analysis based on utility functions. Finally, the hypothesis is related to the gravity and potential concepts that have been used in travel forecasting and in other contexts of regional science.

In a market with hetergeneous products, how is market penetration affected by distance? How does the competitive position of supermarkets, newspapers, shopping centers, recreational facilities, schools, cultural centers, service centers, cities, tourist attractions, national parks, and foreign countries depend on distance? Questions like these cannot be answered by a noncritical application of conventional models of market and supply areas as developed in location theory, since these are based on the assumption of homogeneous goods. One possible way of coping with the effect of distance which is suggested by general social science--or sometimes "social physics"--is to introduce a concept of attraction decreasing with distance along the lines of Newtonian field theory. However, the underlying economic reasoning is often vague and obscure. The purpose of this paper is to state a simple hypothesis concerning the effect of distance on demand and to demonstrate its implications. This will incidentally produce an extension of the range of application for conventional market and supply areas and will serve to give operational meaning to the gravity or potential theory of spatial competition.

1. The Distance Difference Effect

The basic hypothesis of this paper will be formulated in terms of a specific example. Consider two firms having a single plant each at different locations and offering similar but distinct products. Distance will affect communication as well as transportation costs. *Ceteris paribus* relative sales at a given location will depend on the distances from the two firms.[1]

$$(1)\quad \frac{s_1 - s_2}{s_1 + s_2} = F(r_1, r_2)$$

We now postulate the existence of a measure of economic distance $d(r)$ such that the effect of distance on sales can be expressed in terms of the difference of economic distances.

$$(2)\quad \frac{s_1 - s_2}{s_1 + s_2} = f[d(r_2) - d(r_1)]$$

Since distance affects sales adversely, we stipulate that f is a monotonically increasing function. The distance measure d, on the other hand, is a monotonically nondecreasing function of geometric distance. Examples of measure of economic distance which have some empirical relevance are

$d(r) = kr$	proportionality
$d(r)$	a step function ("zonal tariff")
$d(r) = \log r$	a logarithmic measure of geometric distance.

Equation (2) may be rewritten in terms of the sales ratio $\frac{s_1}{s_2}$ or $\frac{s_1}{s_2} = \frac{1+f}{1-f}$

(3) $\frac{s_1}{s_2} = \phi(d(r_2) - d(r_1))$ where $\phi = \frac{1+f}{1-f}$ is again a monotonically increasing function. A slightly more general hypothesis is that the effect of economic distance may be modified by some basic difference of attractiveness which could be due to price differences or quality differences.

(4) $\frac{s_1}{s_2} = \phi(d(r_2) - d(r_1) + a_{12})$

For simplicity we shall also write $d(r_i) = d_i$. Occasionally we may want to represent the differences a_{ij} of attractiveness algebraically as differences of attraction parameters a_i.

$$a_{ij} = a_i - a_j$$

This is possible (in many ways) whenever

$$a_{ij} = -a_{ji}$$
$$a_{ij} + a_{jk} = a_{ik}$$

One solution is $a_1 = 0 \quad a_i = a_{i1} \quad i \neq 1$

for then
$$\begin{aligned} a_{ij} &= a_{i1} + a_{1j} \\ &= a_{i1} - a_{j1} \\ &= a_i - a_j \end{aligned}$$

For reasons of symmetry we have

(5) $\frac{s_1}{s_2} = \phi(d_2 - d_1 + a_{21})$

where $a_{21} = -a_{12}$.

Comparing (5) and (4) we see that

(6) $\phi(-x) = \frac{1}{\phi(x)}$

In particular $\phi(0) = 1$.

Thus the function $\phi(x)$ need be defined only for positive x, its values for negative x being determined by (6) (or vice versa).

2. Market Areas

A more interesting concept than the sales ratio is market share. Let m_i = market share of firm i. In the case of two sellers we have

$$m_1 = \frac{s_1}{s_1 + s_2} = \frac{1}{1 + \frac{s_2}{s_1}} = \frac{1}{1 + \phi\left(\frac{r_2}{r_1}\right)}$$

We now define market area to mean that area where the market share of one firm is $\frac{1}{2}$ or greater. In the case of two firms, the entire region is thereby partitioned into two market areas.

The boundary line between market areas is then that locus where

$$\frac{1}{2} = \frac{1}{1 + \phi\left(\frac{r_2}{r_1}\right)}$$

$$\phi\left(\frac{r_2}{r_1}\right) = 1$$

and so, since ϕ is strictly increasing,

$$d_2 - d_1 + a_{12} = 0.$$

In the absence of differential attractiveness a_{12} it follows that the market boundaries are where

$$d_2 = d_1$$

i.e., where the economic distance from the suppliers are equal. In the case of a strictly monotonic distance measure it follows then that geometric distances are also equal

$$r_2 = r_1$$

and this is the same criterion as for homogeneous products. However, nothing has been said so far on how geometric distance is defined. Of course, any distance measure $r(P_1, P_2)$ must satisfy the usual postulates

(1) $r(P_i, P_j) \geqq 0$

(2) $r(P_i, P_j) = r(P_j, P_i)$

(3) $r(P_i, P_i) = 0$

(4) $r(P_i, P_j) + r(P_j, P_k) \geqq r(P_i, P_k)$.

In location theory it is sometimes useful to consider not Euclidian distance but distance as measured in a rectangular grid of infinite density. Let x_i, y_i denote the Cartesian coordinates of point P_i. $P_i = (x_i, y_i)$, then

(5) $r(P_i, P_j) = |x_i - x_j| + |y_i - y_j|$

When Euclidian distance is used, the market areas of two supplied points are separated by the normal bisector of the line segment joining the two given points. Using the rectangular grid measure for distance, the dividing line consists of at most three line segments with the middle section running at an angle of 45° (Figure I) to the grid.

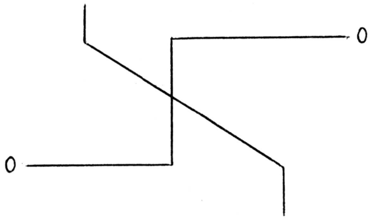

FIGURE I

If attractions differ, $a_{12} \neq 0$, then in a rectangular grid the dividing line of market areas is of the same general shape as Figure 1 but displaced towards the more favored location. In the case of Euclidian distance, the market boundary is the arc of a hyperbola enclosing the less favored location.

The concept of market area may be extended to more than two suppliers. We define the market area of a supplier to be that region where this supplier's market share is dominant, i.e., larger than that of any other supplier. Thus the market area of firm 1 is the set of points for which $m_1 \geq m_i$ for all i. Now market share m_i is defined by

$$m_i = \frac{s_i}{s_1 + \ldots + s_i + \ldots + s_n}$$

or

$$m_i = \frac{1}{\frac{s_1}{s_i} + \ldots + \frac{s_i}{s_i} + \ldots + \frac{s_n}{s_i}}$$

$$= \frac{1}{\phi(d_i - d_1) + \phi(d_i - d_2) + \ldots + \phi(d_i - d_i) + \ldots + \phi(d_i - d_n)}$$

assuming away any differential attractions a_{ij}. The market area of supplier 1 is then where

$$(6) \quad \phi(d_1 - d_1) + \phi(d_1 - d_2) + \ldots + \phi(d_1 - d_n) \leq \phi(d_j - d_1) + \ldots + \phi(d_j - d_n) \quad \text{all } j, \ < \text{ some } k.$$

Consider now the point set where

$$(7) \quad d_1 - d_i \leq d_j - d_i \text{ for all } i.$$

This is the conventional market area of firm 1 as defined for homogeneous commodities. Substitution of (7) in (6) shows that the newly defined market areas (6) include the conventional market area (7). Since the conventional market areas are a partitioning of the entire space--such that any point belongs to at least one market area and every interior point to exactly one market area--the new and conventional market areas must be identical. Thus also in the case of multiple firms, areas of dominant market share may be identified with conventional market areas of nearest supplier.

Of course, all statements about market areas may be translated, _mutatis mutandis_, into equivalent statements about supply areas. Thus Lösch [Lösch] has defined the commuting belt of a city (Einpendlergeibeit) as that territory from which commuters will go predominantly to this city, i.e., as that region where the city's share of the labor market is dominant. [K. Fox and B. Berry have mapped these commuter zones for the U.S.]

The concept of a market area can also be used in an aggregated sense. Thus the aggregate market area or "hinterland" of a city may be defined as that territory where aggregate sales from this city dominate sales from any other city. We may say that this city's aggregate market share dominates that of any other city in this region. This defines again a partitioning which coincides with conventional market areas for the sale of a "representative" single homogeneous product offered in the various cities at the same price.

3. Iso-Share Lines

Of even greater interest than an assignment of locations to market areas of the various firms is an analysis of the way in which market shares vary with distance. For this, a natural starting point is the determination of those loci on which market shares are constant, to be called iso-share lines.

We consider first the case of two sellers. From

$$m_1 = \frac{1}{1 + \phi(d_1 + d_2 + a_{12})}$$

we see that iso-share lines are loci of constant difference of economic distance. If economic distance is a linear function of geometric distance, this clearly implies that iso-share lines are hyperbolas.

Consider next the case where the measure of economic distance is proportional to the logarithm of geometric distance.

$$d_i = a \log r_i$$

In that case,

$$d_1 - d_2 = \text{constant} = a \log \mu \quad \text{(say)}$$

implies

$$\frac{r_1}{r_2} = \mu$$

the iso-share lines are loci on which the distance ratio is constant.

Notice that the locations of the two firms are themselves loci of constant market shares. Depending on the commodity under consideration, it may be appropriate to assume that market share is unity at the home location of the firm. If the commodities are less than perfect substitutes, market shares will be less than one even at the gates of the firm. (Some employees of General Motors for instance drive foreign cars.)

A market share of 1 for $r_1 = 0$ implies

$$1 = \frac{1}{1 + \phi\left(\frac{r_2}{0}\right)} = \frac{1}{1 + \phi(\infty)}$$

or $\phi(\infty) = 0 \qquad \phi(0) = \infty$ in view of (2).

If on the other hand $\phi(\infty) = \alpha$, then the maximal market shares at the firm location is

$$m_{max} = \frac{1}{1 + \alpha} \quad .$$

As we vary μ we obtain the entire family of iso-share lines. Using Euclidian distance, the iso-share line equations are

$$\mu[(x+1)^2 + y^2] = (x-1)^2 + y^2$$

from which

$$(x - \frac{1 + \mu}{1 - \mu})^2 + y^2 = \frac{4\mu}{(1 - \mu)^2} \quad , \quad 0 \leq \mu \leq \infty \quad .$$

(See Figure 2.)

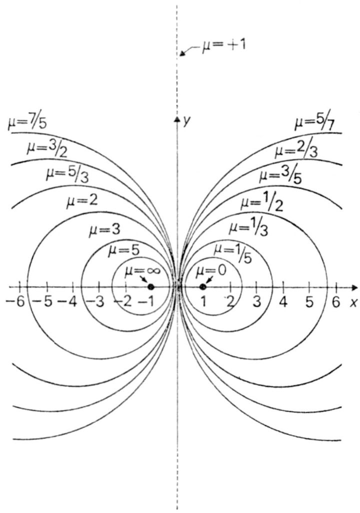

FIGURE 2

For $\mu = \infty$ the iso-share line is the plant location itself $x = +1$, $y = 0$.
For $\mu = 0$ it is the location of the other plant.
For $\mu = 1$ with the center of the circle at infinity, the iso-share line is the y-axis, on which as shown above, share is one half. To each circle in the left half-plane, representing a market share $m_1 > \frac{1}{2}$ there corresponds a symmetric circle in the right-hand plane representing a share of $1 - m_1$. For if

$$\frac{r_2}{r_1} = \mu \text{ then } \frac{r_1}{r_2} = \frac{1}{\mu}$$

and

$$\phi\left(\frac{r_1}{r_2}\right) = \frac{1}{\phi\left(\frac{r_2}{r_1}\right)}$$

from which

$$m\left(\frac{r_2}{r_1}\right) = \frac{1}{1 + \phi\left(\frac{r_1}{r_2}\right)} = \frac{\phi\left(\frac{r_2}{r_1}\right)}{\phi\left(\frac{r_2}{r_1}\right) + 1} = 1 - \frac{1}{1 + \phi\left(\frac{r_2}{r_1}\right)} = 1 - m\left(\frac{r_1}{r_2}\right).$$

The family of iso-share lines is thus a family of circles symmetric to the normal bisector between the two given plant locations.

Next consider the case that economic distance is a step function with a single step. For instance, delivery might be free in a thirty-mile radius. In this case market share assumes at most three values. Consider the three zones defined as follows: (1) the intersection of the free delivery zones, (2) the union of the free delivery zones minus their intersection, (3) the entire region minus the union of the free delivery zones.

At all points of the sets (1) and (3) market shares are constant. They are equal in the absence of differences in product attractiveness. Set (2) consists of two disjoint sets and the share of the firm is larger (but constant) in the set where it is located. These considerations may be extended to the case of a zonal tariff with several zones.

4. Shares and Demand Analysis

Is the sales ratio hypothesis

(4) $\frac{s_1}{s_2} = \phi(d_2 - d_1 + a_{12})$

consistent with conventional demand curves? with a utility function? To answer the latter (which implies the first) let economic distance net of attractiveness be identical with product price p_i at the point of sale, i.e., the delivered price or c.i.f. price

$$d_i - a_i = p_i \ .$$

This is the case if attraction at the mill gate is the negative of the mill price and if economic distance equals transportation cost. Hypothesis (3) reads

(1) $\frac{s_1}{s_2} = \phi(p_2 - p_1)$.

A special utility function that yields a sales ratio function ϕ is the integrated logarithmic function provided we assume that expenditure on commodities 1 and 2 is small enough so that its effect on the marginal utility of money may be disregarded (i.e., the budget constraint may be dropped).

$$\frac{\delta u(s_1, s_2, \dots)}{\delta s_i} = p_i$$

Letting (2) $u = \sum_i (-\int_1^{s_i} \log s\, ds)$ we have

$$\log s_i = -p_i$$

$$\log \frac{s_1}{s_2} = p_2 - p_1$$

$$\frac{s_1}{s_2} = e^{p2 - p1}$$

$$= e^{d_2 - d_1}$$

provided mill prices are equal.[2]

As an example with budget constraint consider the case of a commodity which is costless at the source (water) and has transportation cost porportional to distance. Thus $p_i = r_i$. Consider the logarithmic utility function

(5) $u = \sum_i \log s_i$.

Subject to a budget constraint

(6) $\sum_i p_i s_i = y$

a utility maximizing consumer will buy quantities

$$s_i = \frac{y}{p_i} \ .$$

Thus

$$\frac{s_i}{s_j} = \frac{p_j}{p_i} \ .$$

This is consistent with the hypothesis (1) provided we let economic distance be the logarithm of Euclidian distance.

$$d_i = \log r_i = \log p_i$$

$$\frac{s_i}{s_j} = e^{\log p_j - \log p_i} = e^{d_j - d_i}$$

We conclude that for certain specifications of the functions ϕ and d, the hypothesis (1) is consistent with utility maximization and conventional demand theory, but that in general it is not.

5. The Locational Potential

Consider again the case of many firms. While market areas remain simple, iso-share lines will in general be complicated algebraic or analytic curves defined by equations of the type.

$$(1)\quad m_i = \frac{1}{\phi(d_i-d_1+a_1-a_i) + \phi(d_i-d_2+a_2-a_i) + \ldots + \phi(d_i-d_n+a_n-a_i)} = \text{constant.}$$

For concrete results it is now necessary to specify the functions ϕ. As a first case we consider

$$\phi(x) = e^{-\lambda x}$$

$$\phi(d_i-d_j+a_j-a_i) = e^{\lambda(a_j-d_j) - \lambda(a_i-d_i)}\ .$$

Substituting in (1) and multiplying through with $e^{\lambda(a_i-d_i)}$

$$m_i = \frac{e^{\lambda a_i - \lambda d_i}}{\sum_j e^{\lambda a_j - \lambda d_j}}$$

Writing $e^{\lambda a_i} = A_i$

$$(2)\quad m_i = \frac{A_i e^{-\lambda d_i}}{\sum_j A_j e^{-\lambda d_j}}\ .$$

In the special case where commodity i denoted "travel" to destination i, equation (2) represents a variant of the gravity formula. Since d_i represents economic distance and may be any monotonic function of geographic distance r_i, the general gravity formula is obtained upon substitution of

$$e^{\lambda d(r_i)} = D_i = D(r_i) \quad \text{(say)}$$

into

$$(3)\quad m_i = \frac{A_i/D_i}{\sum_j A_j/D_j}$$

where D_i is another measure of economic distance, sometimes called "resistance." The formula most often applied in travel forecasting uses

$$(4)\quad D(r) = r^{\alpha} \quad \alpha \geqq 1.$$

Substituting (4) in (3) we have

$$m_i = \frac{A_i r_i^{-\alpha}}{\sum_j A_j r_j^{-\alpha}}$$

$$= \frac{1}{\sum_j \frac{A_j}{A_i} \cdot \left(\frac{r_i}{r_j}\right)^\alpha}$$

In the case of equal attractiveness $A_j \equiv 1$ this simplifies to

$$m_i = \frac{1}{\sum_j \left(\frac{r_i}{r_j}\right)^\alpha}$$

$$= \frac{1}{1 + \sum_{j \neq i} \left(\frac{r_i}{r_j}\right)^\alpha}$$

Consider the region where $r_i < r_j$ all $j \neq i$. For large α, market share approximates one in this region and zero outside the region--the case of a homogeneous commodity. Thus α is related to the degree of substitutability of the commodities. In fact, if the commodity's mill price is zero so that delivered price is porportional to distance, then α is the cross elasticity of demand.

More recently exponential functions such as $D(r) = e^{\lambda r}$ or $d_i = r_i$, i.e., economic distance equals geometric distance, have come into use. This approach is sometimes called the "intervening opportunity hypothesis": There is a constant rate of attrition of demand as the commodity moves in an outbound direction.

The denominator of (3) is sometimes called the locational potential. Thus, according to (3) a firm's market share equals its share of the location potential.

So far nothing has been said about the absolute levels of sales at the various locations rather than their relative size. In conclusion we want to indicate briefly how the gravity approach can be turned into a relationship between total sales and "potential."

Assume, as the simplest possibility, that the good is a necessity of which a constant amount is bought per capita. Then

$$s_i = \gamma P m_i$$

where P is population at the location and γ is a constant. Total sales of firm i are then

$$S_i = \int s_i d\omega$$

$$= \lambda \int P m_i \, d\omega$$

$$(5) \qquad = \gamma \int \frac{P(\omega) A_i / D_i(\omega) \, d\omega}{\sum A_k / D_k(\omega)}$$

where ω denotes location. Assume as a first approximation that the "force of competition" as given by the denominator of (5) is nonconstant at all locations ω. This is true when rival plants k are numerous and equally spaced.

$$(6) \quad S_i = \hat{\gamma} A_i \int P(\omega) / D_i(\omega) \, d\omega$$

where $\hat{\gamma}$ is another constant. Here another concept of potential is brought forth, the so-called "potential of population"

$$(7) \quad \pi = \int \frac{P(\omega)}{D_i(\omega)} \, d\omega$$

a measure of the "proximity to masses." The assumption that sales are a function of the potential of population is thus shown to involve the hypothesis

$$s_i(\omega) \sim \frac{P(\omega) A_i}{D_i(\omega)}$$

which is considerably more special than the weak postulate (1.3) on which the analysis of market shares in this paper has been based.

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FOOTNOTES

*An earlier version of this paper was published in the Swedish Journal of Economics--Ekonomisk Tidskrift, 1969, pp. 53-63. The present version considers the implications of the broader hypothesis that the sales ratio depends on the difference of economic distances rather than on the ratio of geometric distances.

[1]A more reasonable assumption than that a relation should exist between distances and the difference of absolute sales since these will depend on the sales level and this level might be influenced bh local conditions. Moreover, absolute sales will decline with distance from all suppliers.

[2]The problem of finding a utility function consistent with hypothesis (1) is that of solving

$$(3) \quad \frac{\delta u}{\delta s_1} - \frac{\delta u}{\delta s_2} = \psi\left(\frac{s_2}{s_1}\right)$$

where ψ is the inverse function of p. (We have set $p_i = \frac{\delta u}{\delta s_i}$.) Symmetry requires

$$(4) \quad \psi\left(\frac{s_2}{s_1}\right) = -\psi\left(\frac{s_1}{s_2}\right), \text{ i.e.,}$$

$$\psi(y) = -\psi\left(\frac{1}{y}\right)$$

corresponding to (1.6). The functions ψ satisfying (4) are given by

$$\psi(y) = g(y) - g\left(\frac{1}{y}\right)$$

where g is arbitrary. Consider the special solutions of (4) obtained by solving

$$\frac{\delta u}{\delta s_1} = g\left(\frac{s_2}{s_1}\right) \qquad \frac{\delta y}{\delta s_2} = g\left(\frac{s_1}{s_2}\right).$$

A solution u exists provided g satisfies the integrability condition

$$\frac{\delta g\left(\frac{s_2}{s_1}\right)}{\delta s_2} = \frac{\delta g\left(\frac{s_1}{s_2}\right)}{\delta s_1} \qquad \text{or}$$

$$\frac{1}{s_1} g'\left(\frac{s_2}{s_1}\right) = \frac{1}{s_2} g'\left(\frac{s_1}{s_2}\right)$$

$$y g'(y) = g'\left(\frac{1}{y}\right).$$

However, not all functions g will generate solutions u that are bona fide utility functions. Thus if g is a power function, the only solution is readily shown to be porportional to

$$u = \sqrt{s_1 s_2} \quad .$$

But this function is homogeneous of degree one making the optimal quantities s_i either zero, or infinite, or undetermined.