FORMAL ALLOCATION MODELS FOR URBAN ACTIVITIES*

Jean Paelinck

Netherlands Economic Institute

Formal allocation models for urban activities

Summary

1. Introduction
2. Formal demand allocators and properties.
3. Allocators for urban activities
4. Implementation of the model
5. Uses of the model

1. Introduction

Empirical studies of urban economic activities are now classical.¹ They relate the absolute amount of employment in given economic sectors or the employment shares, either to total population, or to some basic employment.

No formal consistent model is known to us. On the other hand, spatial characteristics are generally absent; the authors implicitly assume that sufficient space will be available. As we will see, the situation is analogous to that of a consumer confronted only with goods at constant prices.

This paper proposes two allocation models for urban activities; in a second phase of our research we attempted to estimate the parameters of one of them on the basis of available U.S. data.

2. Formal demand allocators and properties

Demand allocators have been intensively studied in recent years.² Their formal properties could be of some use in the analysis of urban activity patterns. Two models will be shortly analyzed hereafter.

2.1 Linear expenditure systems

The linear expenditure system is written as

$$p_i q_i = p_i \bar{q}_i + b_i (d - \sum_j p_j q_j) \quad [1]$$

where

p_i = price of good i ;

q_i = quantity consumed of good i ;

$\bar{q}_i$ = minimal quantity consumed of good i whatever its price
($p_i \bar{q}_i$ = "committed expenditure" on good i);

*The present study was made when the author was visiting the Departments of Economics and of City and Regional Planning (M. I. T.), and was also a member of the Joint Center for Urban Studies of Harvard University and M. I. T.

d = total expenditure at current prices;

$\Sigma_j p_j q_j$ = total committed expenditures ($d - \Sigma_j p_j \bar{q}_j = S$ = "supernumerary expenditures")

b_i = allocator of supernumerary expenditures

$$(\Sigma_i b_i = 1, b_i > 0).$$

One verifies easily the additivity of the system:

$$\Sigma_i p_i q_i \equiv d \quad [2]^3$$

The typical elasticities of the system are:

$$\frac{Eq_i}{Ed} \stackrel{\text{def}}{=} \frac{Ep_i q_i}{Ed} = b_i a_i^{-1} > 0 \quad [3]$$

where a_i is the budget-share of good i , $p_i q_i d^{-1}$.

$$\begin{aligned} \frac{Eq_i}{Ep_i} \stackrel{\text{def}}{=} \frac{Ep_i q_i}{Ep_i} - 1 &= \frac{(1-b_i) \bar{q}_i}{q_i} - 1 \\ \text{with } -1 &< \frac{Eq_i}{Ep_i} < 0 \end{aligned} \quad [4]$$

for normal conditions ($q_i q_i^{-1} < 1$).

$$\frac{Eq_i}{Ep_j} \stackrel{\text{def}}{=} \frac{Ep_i q_i}{Ep_j} = - \frac{b_i p_j \bar{q}_j}{p_j q_j} < 0 \quad [5]^4$$

The Slutsky-elasticities are:

$$\sigma_{ii} = E^*_{ii} a_i^{-1} + E^*_{ii} = a_i^{-1} (1-b_i) \frac{p_i q_i}{p_i q_i} - 1 < 0 \quad [6]$$

$$\sigma_{ij} = E^*_{ij} a_j^{-1} + E^*_{ij} = b_i a_i^{-1} d^{-1} (d - p_j q_j a_j^{-1}) > 0 \quad [7]$$

Equations [4] and [7] show that the system is restricted in that it does not allow for price-elastic and complementary goods.

2.2 Log-linear allocators

The log-linear allocator is written as:

$$p_i q_i = \frac{\alpha_i (p_i d^{-1})^{\beta_i} d}{\sum_j \alpha_j (p_j d^{-1})^{\beta_j}} \quad [8]$$

the variables being the same as defined in 2.1, α_i and β_i being non-constrained parameters. The system is clearly additive. Its typical elasticities are

$$\frac{Eq_i}{Ed} \stackrel{\text{def}}{=} \frac{Ep_i q_i}{Ed} = 1 - \beta_i + E_j a_j \beta_j \quad [9]$$

$$\frac{Eq_i}{Ep_i} \stackrel{\text{def}}{=} \frac{Ep_i q_i}{Ep_i} - 1 = (1 - a_i) \beta_i^{-1} \quad [10]$$

$$\frac{Eq_i}{Ep_j} \stackrel{\text{def}}{=} \frac{Ep_i q_i}{Ep_j} = -a_j \beta_j \quad [11]$$

and the Slutsky-elasticities may be computed as:

$$\sigma_{ii} = \left(\frac{1-a_i}{a_i} \right) [(1-a_i) \beta_i - 1] + \sum_{j \neq i} a_j \beta_j \quad [12]^5$$

$$\sigma_{ij} = 1 - (\beta_i + \beta_j) + \sum_j a_j \beta_j = \frac{Eq_i}{Ed} - \beta_j \geq 0 \quad [13]$$

The only limitation of the model is the equality of all cross-elasticities [11] for given j , ψ_i

3. Allocators for urban activities

The main problem is to find an analog for the price paid by the consumer.

It appears to us that spatial density could play this role in an urban allocator. Models [1] and [8] would then take the form:

$$\frac{E_i S_i}{S_i} \equiv E_i = \frac{E_i}{S_i} \cdot \bar{S}_i + b_i \left(E_j - \sum_j \frac{E_j}{S_j} \cdot \bar{S}_j \right) \quad [14]$$

$$\frac{E_i}{S_i} S_i \equiv E_i = \frac{\alpha_i (E_i S_i^{-1} E^{-1})^i \cdot E}{\sum_j \alpha_j (E_j S_j^{-1} E^{-1})^j \beta_j} \quad [15]$$

where

E_i = sectoral employment;

E = total employment

S_i = surface occupied by activity i

Equations [14] and [15] can be rewritten as

$$D_i S_i \equiv E_i = D_i \bar{S}_i + b_i (E - \sum_j D_j \bar{S}_j) \quad [16]$$

$$D_i S_i \equiv E_i = \frac{\alpha_i (D_i E^{-1})^i \cdot E}{\sum_j \alpha_j (D_j E^{-1})^j \beta_j} \quad [17]$$

Equation [16] appears to be a bit difficult to interpret: a minimal surface, S_i , whatever the density, D_i does not seem to be a very common concept, unless specialists in urban land use can give it some operational meaning. Moreover,

$$\frac{E(E_i) (1 - b_i) \bar{S}_i}{ED_i S_i} > 0$$

so that employment never reacts negatively to density (unless $s_p > 0$, of which the meaning is even less clear).

For these reasons, equation[17] seems to be more adapted. Meaningful elasticities would be:

$$a) \frac{E(S_i)}{E(E)} = 1 - \beta_i + \sum_j a_j \beta_j$$

measuring the relative evolution of sectoral land use, S_i , for a given relative evolution of total employment, E , all densities constant;

$$b) \sigma_{ii} = \frac{(1 - a_i)}{a_i} [(1 - a_i) \beta_i - 1] + \sum_{j \neq i} a_j \beta_j$$

measuring the relative evolution of sectoral land use, S_i , for a given relative evolution of sectoral density, D_i , all other densities constant and total employment being adapted to the new density.

$$c) \sigma_{ij} = \frac{E(S_i)}{E(E)} - \beta_j$$

measuring the relative evolution of sectoral land use, S_i for a given relative evolution of sectoral density of sector D_j , all other densities constant and total employment being adapted to the new density. Sectors i and j would appear to be competing or complementary, with respect to densities according to whether $\sigma_{ij} > \text{or} < 0$.

4. Implementation and uses of the model

In order to implement the model one would need a sectoral breakdown of employment, together with estimates of occupied surfaces, for different cities and/or different years. For self-contained metropolitan areas, housing might be included, $\bar{P}$ (population) substituted for the corresponding E_i , $\bar{S}_H$ measuring housing density.

This program has been implemented as follows.

4.1 Source of data

Land use data are referred to in some classical works on the subject⁷. Tables in Bartholomew (1955) give absolute and percentage of value of land uses in central cities. For 37 of them out of 53,⁸ the following grouping has been made.

<u>Our Categories</u>	<u>Bartholomew's Categories</u>
a. Residential use	aa. Single family dwellings
	ab. Two-family dwellings
	ac. Multi-family dwellings
b. Use by services	ba. Commerical use.
c. Industrial use	ca. Light industry
	cd. Heavy industry
d. Semi-public and public use	d. Semi-public and public use.

Table 1 displays the data in columns (C), (D), (E), (F); columne (B) lists the years of observation. For the occupational side of the picture, use was made of the 1952 and 1949 City and County Data books, except for population

(column G) taken from Bartholomew (1955) and corrected downward for "Institutional population" (included in column J).

The other occupational categories are as listed below. They have all been extrapolated or retropolated on a basis proportional to a linear evolution of total industrial employment ("all employees") between 1939 and 1947; to compute these factors, 1939 "production-workers" figures were blown up on the basis of 1947 ratios between "all employees" and "production workers".

<u>Categories</u>	<u>Content</u>
b. Services	ba. Retail trade: i) Active proprietors of unincorporated businesses. ii) Total paid employees bb. Personal, business and repair services. i) as sub (ba). ii) bc. Hotels (total paid employees) bd. Amusement business (id). be. Finance, Insurance and Real Estate (id). bf. Professorial and related services (id).
c. Industry	ca. Industry (all employees) cb. Wholesale trade (id).
d. Public and semi-public activities	da. Institutional population db. City government dc. City schools i) Teachers ii) Enrollment dd. Hospitals i) Beds ii) Personnel

(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(J)	(K)
TABLE 1. Input Data										
<u>City</u>	<u>Year of Observation or Extrapolation</u>	<u>Land Use (acres)</u>				<u>Population and Employment</u>				<u>Total (G) through (J) (5)</u>
		(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	
1. Baton Rouge, La.	1946	358.66	82.50	81.66	265.61	34,600	13,988	5,413	9,102	63,103
2. Battle Creek, Mich.	1947	1,658.80	130.70	305.30	339.40	43,360	8,121	12,383	14,094	77,957
3. Jefferson City, Mo.	1952	930.80	97.50	77.90	462.90	20,000	4,419	3,014	8,160	35,593
4. Kankakee, Ill.	1945	828.90	58.90	92.20	81.90	22,170	4,446	2,901	8,362	37,879
5. Mason City, Iowa	1940	940.60	56.90	467.60	267.10	25,250	3,542	2,996	5,180	36,968
6. Muskogee, Okl.	1945	1,522.62	107.92	61.90	549.32	37,150	5,369	2,021	10,802	55,342
7. Petersburg, Va.	1943	860.31	49.29	221.57	273.36	30,380	6,505	5,117	16,521	58,523
8. Quincy, Ill.	1945	1,597.50	87.20	214.40	167.30	42,350	5,783	6,072	6,227	60,432
9. Rock Island, Ill.	1940	1,392.50	91.90	221.20	283.40	41,900	4,528	8,572	3,842	58,842
10. Roswell, New Mex.	1946	891.26	69.37	185.64	960.09	20,390	3,263	598	5,892	30,143
11. Santa Fe, New Mex.	1946	815.56	56.98	53.24	604.80	23,600	3,490	800	6,180	34,070
12. West Palm Beach, Fla.	1952	1,614.40	266.00	138.60	274.00	43,750	8,603	2,102	13,623	68,078
13. Bringhamton, N. Y.	1948	1,853.60	288.70	288.70	593.00	82,100	14,429	13,312	26,114	135,955
14. Davenport, Iowa	1945	2,708.00	116.00	218.00	722.00	74,700	11,689	7,997	12,307	106,693
15. Decatur, Ill.	1938	2,004.70	123.30	259.90	195.00	57,350	6,483	7,094	9,618	80,545
16. Greenville, S. C.	1950	2,790.80	250.00	188.20	746.30	57,817	12,465	7,871	12,892	91,045
17. Hamilton, Ohio	1967	1,581.10	95.10	313.80	308.20	54,400	6,986	15,723	9,260	86,369
18. Lansing, Mich.	1937	2,114.60	131.20	542.90	465.30	83,100	7,821	13,456	9,373	113,750
19. Lincoln, Neb.	1951	3,349.00	182.00	208.00	1,064.00	96,700	23,512	12,446	26,432	149,090
20. Portsmouth, Va.	1946	1,091.00	81.00	151.50	324.20	81,925	9,000	2,379	19,020	112,324

(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(J)	(K)
<u>TABLE 1 Cont. Input Data</u>										
<u>City</u>	<u>Year of Observation or Extrapolation</u>	<u>Land Use (acres)</u>				<u>Population and Employment</u>				<u>Total</u>
		(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(G) through (J) (5)
21. Racine, Wis.	1938	1,466.00	163.40	298.30*	293.10	71,950	3,818	9,154	5,584	90,506
22. St. Petersburg, Fla.	1941	2,797.10	185.70	108.70	950.50	60,700	9,537	1,337	9,096	80,670
23. Sioux Falls, S.D.	1949	1,579.40	148.80	269.70	1,371.50	51,135	9,703	7,637	13,176	81,651
24. Schenectady, N.Y.	1945	1,869.90	205.60	554.10	739.00	87,325	11,282	29,679	13,917	142,203
25. Topeka, Kansas	1942	2,577.00	127.50	178.00	313.60	67,500	11,523	5,211	18,491	102,725
26. Corpus Christi, Tex.	1951	4,035.00	458.90	642.90	739.00	110,700	19,059	4,195	26,840	1160,794
27. Des Moines, Iowa	1939	7,523.80	415.90	647.10	2,807.30	152,500	22,232	13,716	25,449	213,897
28. Kansas City, Kansas	1938	3,481.50	216.70	765.80	430.20	124,900	6,792	10,824	15,896	158,421
29. Oklahoma City, Okla.	1945	6,312.65	350.24	639.54	780.01	225,600	36,585	15,563	49,176	326,926
30. Richmond, Va.	1942	5,429.40	412.40	1,149.70	2,064.70	191,000	38,538	32,179	48,160	309,877
31. Utica, N.Y.	1948	2,018.60	278.00	545.00	1,171.20	103,750	16,731	18,618	26,161	165,760
32. Wichita, Kansas	1944	5,324.70	261.70	386.00	971.90	114,200	18,618	12,157	26,936	176,092
33. Dallas, Texas	1944	12,262.15	608.33	796.27	1,424.70	336,500	63,953	47,247	67,122	514,822
34. Dayton, Ohio	1956	6,657.00	811.00	1,081.00	1,403.00	252,000	48,990	106,339	59,522	467,451
35. Memphis, Tenn.	1949	8,745.60	582.40	1,904.90	1,656.10	259,000	29,818	24,309	39,648	352,775
36. Newark, N.J.	1945	2,902.50	791.00	2,058.50	2,209.00	428,750	68,233	104,823	682,294	682,294
37. St. Louis, Missouri	1935	12,323.40	1,735.30	3,138.00	3,485.50	817,000	59,296	992,882	1,042,257	1,042,257

*includes railroad property

One notices that extractive industry and construction have been neglected; transportation (including railway personnel) has been classified under semi-public land used (though railroad property has been neglected in these uses). On the other hand, most of the figures had to be corrected upwards, as information on small numbers was not disclosed; for that very reason, arbitrariness probably never exceeds 1% or the finally obtained figure. Moreover, it is evident that far better decisions would have been taken, had the required figures been available; we tried anyway, on the basis of published figures, to reach as high a degree of homogeneity as possible, before carrying out the analysis proper.

Figures appear in Table 1, columns (G) through (K).

4.2 General shape of the empirical curves

In order to analyze the data in a simple way, the following remarks should be made.

When equation [17] is rewritten stochastically, one has

$$\sum_i D_i S_i \equiv E_i = \frac{\sum_i \alpha_i (D_i E^{-1})^{\beta_i} e^{\gamma_i} \cdot E}{\sum_i \alpha_i (D_i E^{-1})^{\beta_i} e^{\gamma_i}} \equiv E \quad [18]$$

which allows one to compute equation [17] statistically in the "separated" form

$$E_i = \alpha_i (D_i E^{-1})^{\beta_i} e^{\gamma_i} \quad [19]$$

where E_i is a stochastic term exhibiting the ordinary "nice" properties for statistical fitting.

Further, neglecting E_i in [19], and remembering that $D_i = S_i/E_i$, one has

$$\begin{aligned} \log E_i &= \beta_i [\log E_i - \log S_i - \log E] + \alpha_i^* \\ &= B_i (\log S_i + \log E) + A_i \end{aligned}$$

$$\text{where} \quad B_i = \beta_i (1 - \beta_i)^{-1} \quad [20]$$

$$\text{whence} \quad \beta_i = B_i (B_i - 1)^{-1}$$

The quantities $S_i E$ have been computed; graphs 1 to 4 relate them to the corresponding E_i figures.

The graphs should normally have displayed a concavity towards the abscissa; on first inspection this does not seem to be the case, except perhaps in their "upper parts".

To test out whether a systematical factor is at work here, we computed the Kendall rank-correlations¹⁰ of the variables ES_i with the percentage of vacant land¹¹. It appeared to us that a bias in the curve would have been caused by systematic differences in available land for expansion. The results were as follows.

	Kendall's τ ¹²	Significance (normal deviates)
a. Population	-0.2432	
b. Services	-0.2012	-2.4394
c. Industry	-0.2943	-2.0152
d. Public and semi-public activities	-0.1562	-2.9545
		-1.5606

TABLE 2.
Vacant land ratios (in %00)

1. Baton Rouge, La.	2607	19. Lincoln, Nebraska	2979
2. Battle Creek, Mich.	3924	20. Portsmouth, Va.	1941
3. Jefferson City, Mo.	5765	21. Racine, Wis.	3010
4. Kankakee, Ill.	1780	22. St. Petersburg, Fla.	6614
5. Mason City, Iowa	6110	23. Sioux Falls, S.D.	2536
6. Muskogee, Okl.	3082	24. Schenectady, N.Y.	2214
7. Petersburg, Va.	2595	25. Topeka, Kansas	1926
8. Quincy, Ill.	1310	26. Corpus Christi, Tex.	586
9. Rock Island, Ill.	4087	27. Des Moines, Iowa	4799
10. Roswell, New Mex.	2927	28. Kansas City, Kansas	2631
11. Santa Fe, New Mex.	8689	29. Oklahoma City, Okla.	1729
12. West Palm Beach, Fla.	4402	30. Richmond, Va.	3411
13. Brighampton, N.Y.	2971	31. Utica, N.Y.	3381
14. Davenport, Iowa	4286	32. Wichita, Kansas	1853
15. Decatur, Iowa	1850	33. Dallas, Texas	1824
16. Greenville, S.C.	4419	34. Dayton, Ohio	1527
17. Hamilton, Ohio	1167	35. Memphis, Tenn.	3293
18. Lansing, Mich.	2480	36. Newark, N.J.	1657
		37. St. Louis, Mo.	1605

The results are significant at the 5% bilateral level for the first three categories, though τ does not appear to be exceptionally high.

An other remark flows from inspection of the obviously "outlying" points in Graphs 1 through 4. Trendwise we observed that a lower ordinate is correlated with a lower value of vacant land (call this variable r). Systematic exploration of the graphs leads to the following conclusions, where + means a positive deviation from the expected curve for E_i , and a positive deviation from the overall unweighted mean for the r 's ($\bar{r} = 3080$).

Graph	Observation ¹³	Bias	$r - \bar{r}$
1.	Newark, 1945	+	-
2.	Lincoln, 1951	+	-
	Kansas City, 1938	-	-
	Saint Louis, 1938	-	-
3.	Corpus Christi, 1951	-	-
	Kansas City, 1938	-	-
	Des Moines, 1939	-	+
	Oklahoma City, 1945	-	-
	Dayton, 1952	+	-
	Memphis, 1939	-	+
	St. Louis, 1938	-	-

Graph (cont.)	Observation	Bias	$\bar{r}-r$
4.	Oklahoma City, 1935	+	-
	Des Moines, 1939	-	+
	St. Louis, 1938	-	-

For the 14 cases listed, 7 sign patterns correspond, and 7 diverge. Some of these discrepancies might be due to extrapolation (Kansas City, Des Moines, St Louis), but they don't appear systematically in all four **graphs**; moreover, some cities have equally low r 's without appearing systematically in this discrepancy picture (e.g. the r 's for Newark and Saint Louis are nearly the same: 1607 and 1605).

4.3 Regression analysis

Two types of regression analysis have been performed in the light of the previous discussion.¹⁴ On the one hand equation [20] has been fitted by least squares to a set of data not containing the controversial observations. On the other hand an "expanded" equation

$$\log E_i = B_i^* (\log S_i + \log E) + C_i^* \log r + A_i^* \quad [21]$$

has also been fitted by multiple linear regression on the complete sets of data.

The results are as follows:

	Land use type	B_i	$t_{B_i}^{*15}$	C_i^*	$t_{C_i}^{*15}$	r^2, R^2^{16}	F^{17}	d^{18}
[20]	a.	0.5288	29.38	---	---	0.9621	863.49	1.8086
	b.	0.5099	15.59	---	---	0.8836	242.92	1.8732
	c.	0.6944	10.27	---	---	0.7901	105.41	1.8897
	d.	0.4617	9.77	---	---	0.7490	95.48	1.6427
[21]	a.	0.5342	23.94	-0.0783	-1.1855	0.9527	342.17	1.5216
	b.	0.4721	12.86	-0.0505	-0.4388	0.8525	98.23	2.0086
	c.	0.5949	9.41	-0.0793	-0.3790	0.7548	55.28	1.8498
	d.	0.4201	10.7116	-0.3103	-2.6716	0.8063	70.74	1.4387

They show the following characteristics:

- All the B_i^* coefficients are in the range [0.45-0.70], and highly significant.
- Introduction of r does not considerably affect the value of the obtained B_i^* 's.
- The C_i^* coefficients are all negative, and, except for case d, non-significant.¹⁹
- Introduction of r , and using complete data, hardly affects the fit obtained, except in case d, where the fit is improved.

Taking into account the multiple phases through which the data were processed, the results obtained are rather amazing, urban econometrics being generally characterized by low correlation coefficients and non-significant parameters! More refined tests (for non-linearity, e.g.) should be devised; they have not been used at this stage of the investigation.

4.4 Derived parameters

On the basis of the regression results of equation [21] the following parameters were derived; the a_i used are average values.

Coefficient	Type of Land Use			
	a	b	c	d
β_1	-1.1468	-0.8943	-1.4685	-0.7244
$E(E_1)/E(E)$	1.0449	0.7924	1.3666	0.6255
$\sigma_{1\gamma}$				
a	-0.9892	1.9392	2.5134	1.7693
b	---	-17.6180	2.2609	1.5168
c	---	---	-21.4134	2.0910
d	---	---	---	-12.4262

The main results of these calculations are:

- Residential and industrial land uses are more space-elastic than land uses by services (private, semi-public or public)
- All land uses are competitive.
- Industrial land use and land use for private services have large own substitution elasticities, meaning a high sensitivity to land use pressures.

5. Uses of the Model

Let us briefly summarize our findings.

- Density appears as a significant factor in explaining land use patterns; it should be integrated in theoretical models of the city.²⁰
- Land uses are competitive in the central city, at least at the level of aggregation which was used in this study.
- Vacant land is not a significant explicative factor of land use; for public and semi-public uses, it is significant, but appears to be rather a consequence of the observed land-use pattern, than an explaining factor.

The model itself could be used for the following purposes:

- to measure how sectors compete for use of urban land;
- given forecast of E and of the D_i 's, to project the sectorial S_i 's and E_i 's.

It is evident that its operational use depends on more adequate and more disaggregated data.

FOOTNOTES

¹Not quoting the urban base studies, one can think of G. Alexanderson, The Industrial Structure of American Cities (University of Nebraska Press, 1956), I. Morrisset, PAPRSA, Vol. IV, 1958, pp. 238-258; E. Ullman and M. F. Dacey, The Minimum Requirements Approach to the Urban Economic Base, PAPRSA, Vol. VI, 1960, pp. 175-194.

²J. Paelinck, Contribution a l etude de systemes lineaires de depense, (Namur Faculty of Economics, 1964); and: Sensibilite de projections de la demande a moyen terme a des variations des prix relatifs (Paris, Economie Appliquee, 1966, pp. 353-378.

³This property is still valid in a stochastic system.

⁴Negativity is due to the income effect; see equation

⁵ $\sigma_{ii} < 0$ for $\beta_j < 0, \forall j$.

⁶All these elasticities are variable, as they are functions of the a_i .

⁷Bartholomew, H: Urban Land Uses (Cambridge, University Press, 1932); and: Land Uses in American Cities (ibid, 1955); Chapin, F. Stuart Jr.: Urban Land Use Planning (N. Y., Harper, 1957; 2nd ed. Urbana, University of Illinois Press, 1965). We would like to thank our colleague, W. Porter, for having brought this literature to our attention.

⁸This choice has been made on the basis of availability of employment data, only published for cities of 25,000 and over; see hereafter. The cities are listed in table 1 of this study, (A).

⁹Also on the basis of existing and matching employment data.

¹⁰M. G. Kendall, Rank Correlation Methods, London, Griffin, 1955.

¹¹Source: Bartholomew (1955); see table 2.

¹²We recall that $1 \leq \tau \leq 1$.

¹³The description runs from left to right on the graph.

¹⁴They have been processed through the M. I. T. Troll (0) system; Professor Aaron Fleisher kindly provided the necessary computer time.

¹⁵Student's t

¹⁵Student's t

¹⁶Simple or multiple squared correlation coefficient

¹⁷Fisher-Snedecor F-ratio

¹⁸Durbin-Watson's d-statistic (autoregression of disturbances)

¹⁹This is exactly opposite to the results of the t -tests previously reported (section 4.2)

²⁰Cfr. J. Paelinck, Some dynamic urban growth models, paper read at the Copenhagen meeting of the R.S.A., August 1969, (to be published in PAPRSA).