A Test of Base Theory Using Income Indexes Constructed from a State Input-Output Table

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INTRODUCTION

The purpose of this paper is to present a methodology for the construction of income activity indexes on a regional economy. The viewpoint taken is that the essential characteristics presently utilized in the literature in the construction of existing indexes are measurement without theory.[1] This technique of construction will provide a test of regional base theory in terms of output activity indexes.

Activity indexes are composed to interpret the weighted impact resulting from fluctuations in production sectors of relative importance to the region. The weighting and summation cancels positive and negative effects so that a net indication is received as to whether the region under question has improved or fallen from some previous level of performance. The index then serves as a guage for the total economy reflecting weighted growth, consistency, or stagnation of divers production sectors.

An argument can be presented that an activity index is redundant since Business Week[2] and Survey of Current Business[3] publish periodic personal income estimates. Additionally, several states possess econometric models to predict levels of performance for the economy. Two points can be offered as a counter-argument. First, the activity index does not measure similar variables as the income estimates or econometric regression estimates. The purpose of the index is to gauge the weighted impact of diverse production fluctuations. This concept includes the multiplier effect. The personal income or gross state product estimates are cross-sectional views at a point in time of the impact the index is measuring. Secondly, using the survivor's technique, there appears to be a need for the index. Activity indexes at the national and regional levels have existed together with GNP accounts for a number of years. One has not canceled the other.

There are three problems which arise in the construction of this index. The first concerns the selection of the basket of sectors since not all production categories can be included. Those chosen must be representative of the output movements of the total region. The existing literature reveals that this selection is often an *a priori* judgment sample. Adelman[4] contends that this procedure can cause sampling error to create a variance between two similar indexes constructed for the same region.

The second problem involves the determination of the weights assigned to each sector that enters the index. The literature shows that a wide range of weights are used including value added, percent of participation income, subjective decisions, and no weight at all.[5]

The third problem concerns the choice of a periodic measureable series to approximate the production of each sector definition. If the sector chosen exists empirically as defined, no problem arises. Proxies such as payrolls, crude oil runs, life insurance sales, sales taxes, and employment totals have been substituted for production totals of categories which exist by definition only.

The techniques presented here have several unique festures in contrast to the existing methods of construction cited in the literature, First, two activity Laspeyres indexes are constructed as opposed to the usual one index number theory. One index measures the most volatile or leading output indicators of the region, called the basic sectors. The other measures the coincidental or status quo output indicators, called the nonbasic sector. These two together provide an empirical test of export base theory for the region thus creating a theoretical environment for their existence. Leads and lags and the base ratio between the two were estimated.

Second, a regional input-output table was used for the selection of the basket of sectors. This tool has not been utilized for this purpose at the regional level. Therefore, it presents a substantial improvement in the con-

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stant search for objective defendable criteria to be applied in the selection of sectors most representative of output activity.

Third,the weights assigned to each series were determined from a simultaneous equation solution constructed from the multipliers taken from a regional input-output table. They are interpreted as representative multipliers of any fluctuations in the output sectors. Considered together, these three points provide a solution to the first two problems of construction.

INDEX WEIGHTS

The weighting procedure dictated the number of sectors and their numerical weights for the Laspeyres indexes of the form,

$$\sum \frac{Pi\ qo}{po\ qo} \quad 100$$

where qo = base output weight

Po = base output dollar value

Pi = current output dollar value

Consider a set of simultaneous equations that describe the various conclusions derived from all input-output matrices in a usual, typical interindustry study. These equations may be written in the following from for a 33 x 33 processing sector:

$$y_1 = a_1x_1 + a_2x_2 + \dots + a_{33}x_{33}$$

$$y_2 = b_1x_1 + b_2x_2 + \dots + b_{33}x_{33}$$

$$\vdots \qquad\qquad \vdots$$

$$y_9 = i_1x_1 + i_2x_2 + \dots i_{33}x_{33}$$

The first equation represents the total direct income effect. Each x_i refers to a specific sector in the processing matrix. Each coefficient, a_i, stands for the direct income effect in that respective sector. The value of y represents the total direct income effect shown in the economy or the sum of all thirty-three a's. Since y is the summation of the a's, the value for each x_i in the first equation must equal one. This latter point is consistent with the interpretation given to the direct income multiplier.

The second equation represents the total indirect income effect. Each x_i refers again to a specific sector in the processing sector. Each coefficient, b_i, stands for the indirect effect in that respective sector. The value for y represents the total indirect effect shown in the economy or the sum of all thirty-three b's. The value for each x_i would again be equal to one.

The other seven equations represent respectively: the direct and indirect multipliers; induced multipliers; direct, indirect, and induced multipliers; type one multipliers; type two multipliers; first type sector multipliers; and the second type of sector multipliers. Each x_i refers to a particular production sector in the processing matrix. Each coefficient represents the respective multiplier for that sector. Every y stands for the sum of the coefficients in the equation. Each x, then in each equation equals one.

The set of nine equations does nothing more than express the nine methods used to view changes in income or output from an input-output table. Each coefficient is consistent with the classical interpretation given to that specific multiplier. Each partial derivative with respect to each sector becomes the respective multiplier from the input-output table.

The maximum number of independent equations from the set of nine appears to be five.[6] These five represent, then, only the five multiplier concepts which are primary conclusions from the interpretation of the value added sector and the inverse matrices. They are the equations for direct income; direct and indirect income; direct, indirect, and induced income; sector output multiplier of the first type; and the sector output multiplier of the second type.

To have a solution to thesf five equations that is unique, the system must be consistent and the determinant not equal to zero. Therefore, the number of equations and unknowns was reduced to the following form. Each equation represents one of the multiplier concepts described above:

$$y_1 = a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + a_5x_5$$

$$y_2 = b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_5x_5$$

$$y_3 = c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 + c_5x_5$$

$$y_4 = d_1x_1 + d_2x_2 + d_3x_3 + d_4x_4 + d_5x_5$$

$$y_5 = e_1x_1 + e_2x_2 + e_3x_3 + e_4x_4 + e_5x_5$$

Note that the values for x in the solution will no longer be equal to one. Solving this set of equations will give a value for each processing sector ($x_1 \ldots x_5$) which corresponds to its relative weighted effect upon income or output. Each multiplier or coefficient has served as a measurement of impact upon income in determing the final solution.

Therefore, this paper proposes that the maximum number of five linear independent equations or views upon changes in income dictates that only five processing sectors be utilized. The solution of these five equations simultaneously provides a weight for each series. This solution or weight for a particular processing sector becomes its weight in the final composite activity index.[7]

Earlier it was stated that two composite indexes for output will be formulated. One will represent the coincidental sectors. Therefore, the weighting system dictates that a maximum of five sectors be selected for each index.

It also dictates that five weights must be derived for the leader and five weights for the coincidental. Those five sectors selected as leaders are written into the identical form of the preceding five equations and set equal to the y's ($y_1 \ldots y_5$). A simultaneous solution is derived. One set of five corresponds to the five sectors already designated as leaders, and one set of five corresponds to those already designated as coincidentals.

BASIC SECTORS AND THEIR WEIGHTS

The five industrial categories chosen are those that provide the largest absolute totals of sales to Exports and the Federal Government. To determine these five leader sectors for the State of Missouri, the two columns of Exports and Sales to the Federal Government in the final demand sector of the State Input-Output Table[8] were summed. The five largest absolute totals were chosen. Table 1 shows these sectors and their rank. The relative importance of each category with respect to total output of that sector and with respect to sales within the state is also shown.

Transportation Equipment (TE) ranked first by a large margin over Wholesale Trade Services (WTS), ranked second. Total sales outside the state by TE accounts for 82.5% of the total output as compared to 63.2% for WTS. Chemicals (Ch) ranks third with almost three-fourths of its output being sold to exogenous sources. Transportation Services (TS) ranks fourth and Other Food Products (OFP) rands fifth, selling 41.9% and 58.4%,respectively, to sources outside the state.

Weights for the Leader Output Index

Once five industrial sectors have been chosen to be included within the leader composite index, the weights assigned to each sector can be determined. Each weight is derived from the solution to a set of five linearly independent equations.

TABLE 1.

TOTAL OF EXPORTS AND SALES TO THE FEDERAL GOVERNMENT BY SECTOR

Sector	Input-Output Table Number	Exports and Sales to the Fed. Gov't (000's)	Rank of Exports and Sales to Fed. Gov't	Exports and Sales as % of TP of sector	Exports and Sales as % of sales within state by sector
Transportation Equipment (TE)	21	$2,519,125	1	82.5	471.6
Wholesale Trade Services (WTS)	25	842,356	2	63.2	171.6
Chemicals (Ch)	13	579,529	3	73.5	277.3
Transportation Services (TS)	24	571,750	4	41.9	72.2
Other Food Products (OFP)	7	476,378	5	58.4	140.1

Source: Addition of columns 39 and 40 in Table I of Floyd Harmston, _Missouri Economy Study: An Inter-sectoral Analysis of the Missouri Economy_ (Columbia: University of Missouri School of Business Research Center, 1968), p. 6.

The equations are expressed in the following form:

$$11.3121 = .1694TE + .4817WTS + .2391Ch + .3949TS + .23560FP$$

$$49.7323 = 1.1191TE + 1.2694WTS + 1.3247Ch + 1.3196TS + 1.59280FP$$

$$80.4185 = 1.4647TE + 2.2891WTS + 1.9034Ch + 2.1650TS + 2.30300FP$$

$$49.7921 = 1.0365TE + 1.4093WTS + 1.2663Ch + 1.4750TS + 1.16230FP$$

$$101.2976 = 1.4564TE + 2.0507WTS + 1.7553Ch + 2.8323TS + 2.48660FP$$

The first equation represents the direct income change (multiplier). Each coefficient for each of the five sectors was found by dividing total purchases from the Household sector by total purchases of that sector. The amount, 11.3121, represents the sum of all coefficients for all 34 sectors in the processing matrix.

The second equation represents the direct and indirect income change (multiplier). Each coefficient is the multiplier for that respective sector. The total, 49.7323, is the summ of all coefficients for the 34 sectors. The solution to this set of five equations is shown in Table 2.

TABLE 2.

WEIGHTS FOR THE COMPOSITE INDEXES

Output-Leader		Output-Coincidental	
TE =	- 51.18582153	RTS =	-127.24186707
WTS =	- 16.24691772	RER =	5.85124207
Ch =	51.35717773	FI =	76.44947815
TS =	33.61018372	C =	- 14.94961548
OFP =	9.57964134	PHK =	82.23959351

Source: Derived from data constructed by the author.

NONBASIC SECTORS AND THEIR WEIGHTS

Coincidental sectors are defined as those industrial categories which illustrate the largest absolute totals of Sales Within the State (SWS). SWS is defined as total output of sector i minus the sum of Exports and Sales to the Federal Government of sector i.

To find these five coincidental sectors as described by the weighting system, SWS was calculated for every industrial sector in the Missouri Input-Output Table. Table 3. illustrates the five largest absolute values.

Retail Trade Services (RTS) ranks first with 91.7 percent of this category's total output selling within the state. Real Estate and Rentals (RER) ranks second showing a larger 93.2 percent of output sales within Missouri. Finance and Insurance (FI) is third in absolute total of SWS but is the lowest percentage of SWS to total output with 72.3. Construction (C) ranks fourth in SWS and shows a percentage of 91.4. Professional, Health, and Kindred (PHK) ranks fifth in SWS and shows a percentage of 87.8.

Weights for the Nonbasic Sector

In a similar manner as the leader index, the weights for the coincidental sectors need to be derived once and nonbasic industries have been identified. The five equations used are:

$$11.3121 = .5061RTS + .4978RER + .2233FI + .4170C + .7534PHK$$

$$49.7323 = 1.2201RTS + 1.2658RER + 1.4885FI + 1.5474C + 1.3000PHK$$

$$80.4185 = 1.1521RTS + 1.4396RER + 1.6020FI + 1.4839C + 1.0661PHK$$

$$101.2976 = 4.1943RTS + 4.5782RER + 3.6566FI + 2.4914C + 4.4492PHK$$

The coefficients in these equations have similar interpretations as those used to derive the weights for the leader index. The first equation represents the direct income change. Each coefficient is the multiplier for that sector. The figure, 11.3121, is the sum of the 34 sectors in the processing matrix.

The other four equations represent: the direct and indirect income change; the direct, indirect, and induced income change; the type I sector output multiplier; and the type II sector output multiplier, respectively. Each coefficient is the multiplier for that sector. Each sum to the left of the equal sign is the sum of all sectors included within the processing matrix. The solution to this set is shown in Table 2.

CURRENT DATA

The weighting system for the two indexes has derived two sets of weights in terms of dollars. Inclusion of total output of each sector per quarter into the index for p_i would give a weighted value for total output, once multiplied by each sector's weight.

Since total production and sales tax collections per sector do not exist, the next proxy for total output would be payrolls. Therefore, payroll data were gathered corresponding to the ten series in the leader and coincidental output indexes from the first quarter of 1960 to the first quarter of 1971.[9] Forty-five observations were considered a time period of sufficient duration to test and examine the methodology proposed here.

The two output indexes were constructed once using only payrolls for workers covered by employment security. The coincidental index was constructed a second time utilizing payroll data and two other output proxies--

TABLE 3.

SALES WITHIN THE STATE BY SECTOR

Sector	Input-Output Table Number	Rank of Sales Within the State	Total Sales Within State (000's)	Exports and Sales to Fed. Gov't as a % of Sales Within the State	Sales Within State as a % of TP
Retail Trade Services (RTS)	26	1	$2,135,206	9.1	91.7
Real Estate & Rentals (RER)	34	2	1,861,997	7.3	93.2
Finance & Insurance (FI)	32	3	1,153,672	39.4	71.3
Construction (C)	33	4	964,522	9.4	91.4
Professional Health & Kindred (PHK)	27	5	776,393	13.9	87.8

Source: All figures taken from Table 7 of Floyd Harmston, Missouri Economy Study: An Intersectoral Analysis of the Missouri Economy (Columbia: University of Missouri School of Business Research Center, 1968), p. 6-7.

retail sales and the value of construction permits. Total retail sales for the state were used instead of payrolls as an approximation for the output level of Retail Trade Services, and the value of construction permits for the state was used instead of payrolls as an approximation for the output level of Construction.[10] The coincidental index using all payrolls and the one utilizing retail sales and construction values in addition to payrolls were compared.

Smoothing and Deflation of Data

The only editing procedures performed on the raw data were the use of a price deflator and experimentation with smoothing using moving averages over varying quarters. A definite attempt was made to leave the data in their original form.

All payroll data, and retail sales and value of construction permits where used, were deflated by the national Consumer Price Index (CPI) with a base of 1967. Raw figures for a specific quarter were deflated by an average CPI for the three months composing that quarter.

The deflated data were subjected to a moving average varying from one to twelve quarters. Moving quarters for an even number of quarters were centered.

The base value used in all indexes was July, 1967. This period was chosen to correspond with the base of the CPI and latest revisions of bases at the federal level.

LEADS-LAGS AND THE PROPORTION OF THE BASIC TO THE NONBASIC SECTOR

An estimate of time in leads and lags and magnitude of size between the basic and nonbasic sectors were calculated. Following an approach presented by Fishman,[11] consider two strictly stationary sequences, x and y. The means are

$$E(X_t) = U_x, \; E(Y_t) = u_y$$

with autocovariance functions

$$R_{x,L} = E[(X_t - u_x)\,(X_{t+L} - u_x)]$$

$$R_{y,L} = E[(Y_t - u_x)\,(Y_{t+L} - u_x)]$$

and covariance function

$$R_{xy,L} = E[(X_t - u_x)\,(Y_{t+L} - U_x)]$$

with the crosscorrelation function

$$\rho_{xy,L} = R_{xy,L}/(R_{x,o}R_{y,o})^{1/2}$$

where $|\rho_{xy,L}| \leq 1.$

The crosscorrelation function will calculate the amount of linear correlation between X and Y for varying time lags. The value of L which causes rho to assume its maximum identifies the time lag with the greatest amount of correlation between X and Y. A positive value is interpreted as a net influence from X to Y. A negative value indicates a net influence in the opposite direction.

This model was applied to the leader and coincidental indexes for all moving averages. The leader output index was considered as a stimulus to each of the coincidental indexes. Table 4. shows the maximum values of rho. The period and the time lead in quarters which produced the maximum are listed.

The maximum crosscorrelation or rho value occurred between the leader output index and the coincidental output constructed around payrolls. A moving average including five periods were necessary to achieve this value.

TABLE 4.

LEAD ESTIMATES OF THE LEADER INDEX UPON THE COINCIDENTAL INDEXES AND THE IMPACT OF THEIR FLUCTUATIONS

Index	Coincidental Output Index with Retail Sales and Value of Construction Permits	Coincidental Output Index with Payrolls
Leader Output Index	$\rho = + .906$	$\rho = + .927$
	Period 12	Period 5
	Lagged 1 Quarter	Lagged 5 Quarters
	$\overline{X} = 1.040$	$\overline{X} = 1.275$
	$\sigma^2 = .001354$	$\sigma^2 = .1238$

Note: The term "lagged" refers to the number of quarters the coincidental index fell behind the leader index in order to achieve the maximum value of rho.

The coincidental lagged the leader by five quarters. The stimulus-response was faster with a coincidental index constructed from payrolls than one constructed with payrolls and retail sales plus the value of construction permits.

Figure 1. illustrates the trend for all three indexes constructed with a filter of five periods. The leader index increased from the early 1960's to a peak in mid-1967. A decline is evident from that peak to 1971. The coincidental index gradually increased throughout the complete time period. A reduced growth pattern is noticeable, however, initiating with the second to third quarter of 1968.

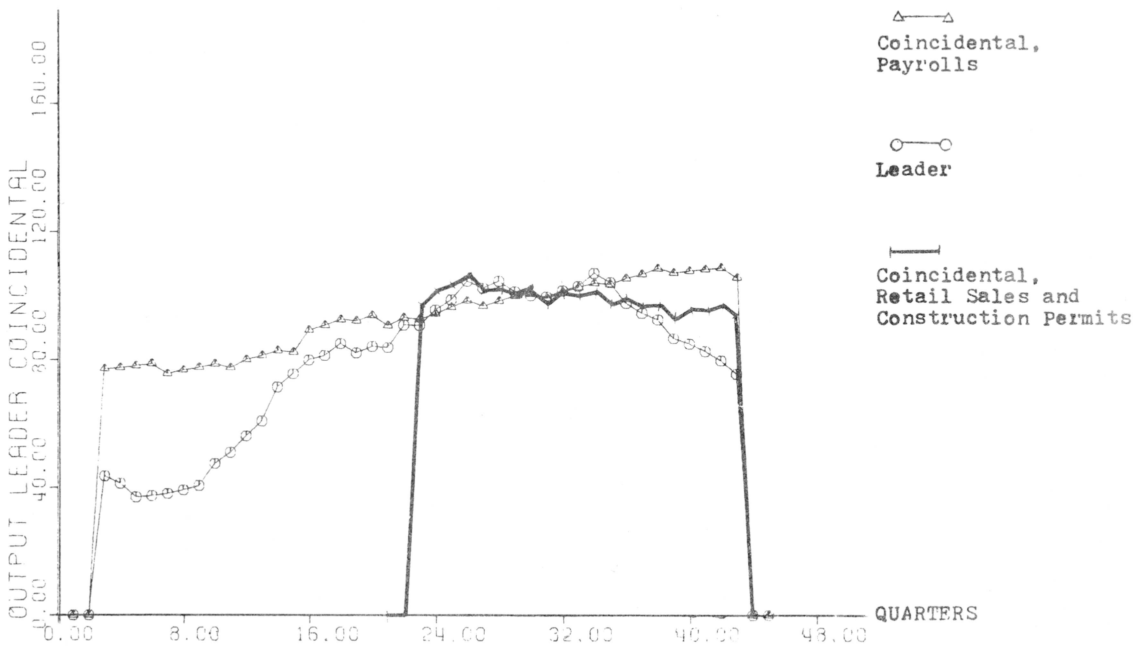
The coincidental output index including retail sales and the value of construction permits fluctuated to a peak in mid-1966. A very gradual decline is evident from that peak in 1971.

The value of $\bar{x}$ is the mean of all coincidental/leader ratios for the period in which the maximum rho occurs. The term ρ^2 is the variance for this mean. A one unit fluctuation in the leader output will cause a 1.275 change with retail sales and construction permits. The smaller variance of the latter means that the precision of the mean is higher.

CONCLUSIONS

The first and most important conclusion is the presentation of a theoretical basis encompassing the construction of activity indexes. This procedure utilized traditional tools of input-output, a Laspeyres index, and base theory. The development of two indexes as opposed to one allows an estimate of the stimulus-response between the basic and non-basic sectors. Five periods and a lag of five quarters was necessary to acheive the maximum crosscorrelation. The base ratio may also be calculated by this dicotomy. The ratio of 1.275 was found to exist at the point the rho value achieved its maximum.

FIGURE 1. OUTPUT INDEXES FOR PERIOD 5



FOOTNOTES

[1]For a survey of the literature, see Donald R. Epley, "A Test of Base Theory Using Income and Employment Indexes Constructed from a State Input-Output Table," (Ph.D. dissertation, The University of Missouri-Columbia, May 1972), pp. 13-50.

[2]Business Week. Business Week's Measure of Personal Income (New York; McGraw-Hill Book Co., Inc., Monthly).

[3]U.S. Department of Commerce. Survey of Current Business (Washington: Bureau of Economic Analysis, Monthly).

[4]I. Adelman, "An Approach to the Construction of Index Numbers," Review of Economics and Statistics, Vol. 40 (August, 1958), pp. 240-249.

[5]Epley, Op. cit., pp. 71-72.

[6]The five equations selected may not represent the only set of linearly independent equations from the nine given. They do represent the traditional, classical results from a standard input-output table and become, therefore, the most important.

[7]Mathematically, a negative solution is acceptable. Economically, a negative weight is contrary to out ex ante expectation. A negative weight here will result, ceteris paribus, in a decrease in the composite index level if the output of a sector should increase, or vice-versa.

The explanation and justification for this result lies in the total variance for all five sectors. A negative weight is said to contribute negatively to the total variance of the solution. Therefore, for proper interpretation, all five composite sectors should be analyzed in the aggregate or index form, not individually.

A similar argument is given for the payment of economic rent. The payments are based upon a functional distribution, and no explanation is offered as to the identity of the recipient. These expenditures must be paid in order for the proper allocation of resources to take place within this market and related ones.

No unique general description of the solutions in terms of the coefficients exists. The Jordan canonical form presents a unique solution for a given matrix only. Furthermore, the description of the solutions in terms of the coefficients will vary from given matrix to given matrix. Additionally, the solutions for a given matrix in terms of its coefficients is acceptable only for a specific order of the equations. A change in the order can change the description of the solutions.

[8]Floyd Harmston, Missouri Economy Study: An Inter-Sectoral Analysis of the Missouri Economy, 1963 (Columbia: University of Missouri School of Business Research Center, 1968), pp. 6-7.

[9]Division of Employment Security, ES-202 Report of Coverage and Contributions (Jefferson City: Missouri State Employment Service, Issued Quarterly).

[10]Retail sales were taken from Bureau of the Census, Monthly Retail Trade (Washington: U.S. Department of Commerce, Monthly), Table 6; and the value of construction permits were taken from F. W. Dodge Construction Statistics, Region VI Bulletin (New York: McGraw-Hill Book Co., Inc., Monthly).

[11]George Fishman, Spectral Methods in Econometrics (Cambridge: Harvard Press, 1969), pp. 11-69.