Urban Structure And The Spacing Of Cities: An Empirical Investigation

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ABSTRACT

There is both empirical and theoretical evidence that the population size of urban centers is closely related to their spatial distribution. Population size involves considerably more than Christaller's concept of centrality: it includes basic and non-basic activity at the secondary, tertiary, and quaternary levels. Concepts based on transportation and administration principles and aspects of Loschian theory explain part of this relationship. To investigate the relationship between specific aspects of functional size and the distance separating cities, an exploratory model is tested in west-central Oklahoma. Ten orthogonal components explain most of the variance in a 35-variable matrix which embraces functional, demographic, social and surface characteristics of the urban system. Four distinct hierarchical levels exist in this area. Scores on the ten components are used to predict the distance separating cities from their nearest neighbors of similar or higher hierarchical levels. Almost 79 percent of the spacing variation is explained, with 68 percent of the total explained by two service-type components.

Central place theory is generally regarded as the only comprehensive model of city size and distribution patterns. Since the extension of the Christaller-Losch theory by Isard (1956) and Berry and Garrison (1958), most analytic studies of the spatial distribution of cities have been developed within its framework. The theory has been used, and subsequently discarded, as a model of intra-urban tertiary activity (Berry, 1967, 1971), and as a framework for testing growth pole theory (Odland, *et al.*, 1973), and it is presently undergoing theoretical reformulations to incorporate a dynamic aspect (White, 1974). Still, it is the theory of urban size-spatial relationships which is of most importance to geographers.

The purpose of the present research is to propose and test a more comprehensive model of city spacing than has hitherto been derived. We show that there is a theoretical basis for examining the relationship between several economic sectors of an urban system and its spatial characteristics. The model derives basic dimensions of an urban system

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via principal components analysis, and relates these dimensions to the distances separating cities, within a hierarchical framework.

Previous Research

Analyses of the spacing of urban centers have been operationalized in several ways. In one approach, groups of cities have been treated as dimensionless points on featureless surfaces, and analyzed with nearest-neighbor techniques devised to identify regularity and randomness, and to assess the extent to which each exists through measures of system entropy.[1] Alternatively, surface disparities may be controlled through topological transformations in attempts to approximate the isotropic surface of classical central place theory (Rushton, 1972). These approaches have a number of drawbacks. Some ignore the size of the cities under analysis, thereby removing one important aspect of central place theory. Others depend on the accurate identification of hierarchical levels — although this is quite difficult to achieve — and different results have been found using different techniques.[2] Moreover, all of the techniques are based on the triangular lattice of Christaller's K=3 system. Theoretically, this pattern will emerge only through the exclusive operation of the marketing principle, although it is doubtful that this exists in isolation anywhere at present. Concepts based on transportation and administration principles and the Loschian structure have not been incorporated in these studies.

Another approach to distinguishing spacing regularities is that proposed by Thomas (1961) in his "expanded central place model." Thomas hypothesized a positive relationship between population size and the distance separating settlements of similar size, treating both variables as continuous rather than discrete. The model was operationalized through linear regression of the form: $d = L(s)$ where d is distance, s is population size, and L is a linear function with positive slope. This was subsequently extended by King (1961) to include other characteristics of the urban centers and aspects of the surface under analysis, in attempts to isolate "noise" from the model. A later test in Ireland also showed promising results (O'Farrell, 1970).

However, although the model is described as an "expanded central place" framework, it is not a test of classical central place theory. Relaxing the assumptions of discrete hierarchical levels and "typical" distances is necessary for testing the relationship, and is more in accord with reality. The independent variable, population size of settlements, was not suggested as a surrogate for centrality, but rather as a heuristic extension of the model. Christaller (1966, p. 17) was careful to distinguish between a city's population size and its centrality, or basic tertiary activity. Several indices have been developed to measure this, including Christaller's (1966, p. 147) telephone index.[3]

Population size includes considerably more than Christaller suggested: even in cities which function solely as service centers, it includes that

proportion of the population engaged in non-basic activity, and in most cities it includes persons engaged in the secondary and quaternary sectors, and possibly reverse commuters working at primary activity, as well as the aged and unemployed. However, Thomas' model empirically validates the relation between an aggregate measure of all sectors of the economy of cities and the distance separating them.

Urban Structure and Spacing: An Exploratory Model

While Christaller provides a theoretical basis for anticipating a relationship between the distance separating cities and the population employed at basic service activity in them, there is also a rationale for including the work force in transportation and administration in the independent variable. Given the relaxation of the rigid hierarchical and geometric structure used by Thomas, there is certainly a relationship between these sectors and spacing in the $K=4$ and $K=7$ systems. Moreover, many writers have suggested that the Loschian pattern is more suited to secondary activities than to tertiary ones (von Boventer, 1963,) so there may also be some tenuous relationship between distance and certain industries which locate independently of functional agglomerative forces. This argument has been recently extended by Parr (1973). He argues that classical "central place theory can be concerned with economic activity of an intermediate type such as wholesaling, transportation, specialized banking and finance", and with certain types of manufacturing activity. The crucial restriction, according to Parr, is that this activity be of a "market-oriented or market-sensitive nature." City location, as King (1961) noted, does not occur on an isotropic surface, and any predictive model must incorporate real-life variations in population density, income, primary industry, and other variables which may affect the spacing of cities. Thus, one might visualize an additive model which embraces both the different aspects of population size and different types of surface. Christaller has shown, under simplifying assumptions, that a triangular lattice of service centers will develop on an isotropic income and density surface. Variations in these surface variables may be expected to affect the spacing of the *service* component. Similarly, variations in transport network density should affect the spacing of the transportation sector of the economy of cities, and the distribution of raw materials will influence the secondary sector

Our approach is to examine the relationship between the various components of population size and surface variations and the distance separating settlements. Past studies have shown that urban systems may be decomposed into a number of basic additive dimensions. According to Ahmad (1965) "it seems that five factors — size, the specific set of economic specialties, population change, density, and some social characteristics such as income and literacy — are sure to emerge as independent differentiating elements of urban systems in any situation."

Thus, although any urban system comprises many economic sectors, dynamic aspects, locational influences and social connotations,[4] the number of underlying dimensions is relatively small.

The present study will decompose a system of cities into its basic dimensions. These dimensions should provide a comprehensive statement of the system's economic specialties, dynamic and locational orientations in relation to the surface on which it is located — which should explain the spatial distribution of the constituent cities. To test this, a hierarchy will first be derived by cluster analysis. Then, within a hierarchical framework, the basic dimensions will be used as predictors for the distance separating cities of the system. This should provide a clearer insight into the nature of city spacing than has hitherto been achieved, and, if successful, would permit the development of inductive theory.

The Operational Model

The 35 variables used as indices of size and surface composition are shown in Table 1. These are broadly related to several different concepts: population size and growth, social aspects, functional emphases, and location. An additional concept is concerned with relevant surface characteristics: these variables were measured at county level for each of the cities under analysis.

In choosing a study area in which to test the model, a city system, or subsystem, is an important requirement. This is to ensure that the spatial distribution of cities under study has developed independently, without undue influence from external forces. Ideally, a closed system, such as an island, is required, since, in an arbitrarily-defined study area, a city's nearest neighbor may lie outside the limits of the area, and distance to another city within the area may give misleading results. In the present study, a system of cities was identified in west central Oklahoma in the United States (Figure 1), using the maps prepared by Berry (1967). In terms of commuting patterns, urban fields, and functional orientation, this area is relatively self-contained, with very little overlap with nearby states. This area was treated as a closed subsystem for purposes of the analysis.

All cities and urbanized areas of not less than 2,500 population identified in the 1970 census were included in the study. This totalled 73 cities, ranging in size from Stroud (2,502) to the urbanized area of Oklahoma City (579,788). The mean population size was 22,925. The area thus identified avoids extreme complexity or simplicity, but it does include an interesting range of growth and functional characteristics.

The Latent Dimensions of the Oklahoma Urban System

Principal components analysis was used to decompose the independent variables into their basic dimensions. This both reduces the number of

TABLE I. THE VARIABLES AND THEIR CONCEPTUAL BASES

Absolute Size	1. Population 1970
	2. Labor force 1970
Growth	3. 1970 population as percentage 1960 population
Social	4. Percent labor force female
Correlates	5. Percent under poverty level
	6. Per capita income
	7. Percent under 18 years old
	8. Percent aged 65 years and over
	9. Fertility ratio
	10. Persons per household
Marketing	11. Percent labor force in wholesale and retail trade
Principle	12. Percent labor force in professional services
	13. Total retail establishments
	14. Annual retail sales
	15. Total food stores
	16. Total furniture, home furnishings, and equipment stores
Transportation	17. Percent labor force in transportation
Principle	18. Scheduled airline stops
	19. Distance to highway
Administration	20. Percent labor force in public administration
Principle	21. County seat
Industrial and	22. Percent labor force in manufacturing
Resource Towns	23. Percent labor force in communications, utilities, and sanitary services
	24. Percent labor force in finance, insurance, business, and repair
	25. Percent labor force in educational services
	26. Percent 3-34 year olds in school
	27. Percent labor force in construction
Locational	28. Number of towns (over 1,000) within 25 miles
Characteristics	29. Distance to a central city of over 50,000
Surface	30. Percent county population rural
Characteristics	31. Per capita county rural income
	32. County rural population density
	33. Percent county rural labor force in primary industry
	34. Percent county area in farms
	35. Average county farm size

variables and eliminates "noise" from the system by isolating the variance-maximizing components. Ten components with eigenvalues greater than unity were rotated to orthogonal simple structure according to the normal Varimax criterion. Together, these ten components accounted for almost 83 percent of the original total variance, with the first four factors accounting for over half the original. Communalities, loadings greater than .4 and less than —.4, and component variances after rotation are shown in Table 2.

All of the components are interpretable in the context of the model to be tested. The first is a central services factor with very high positive
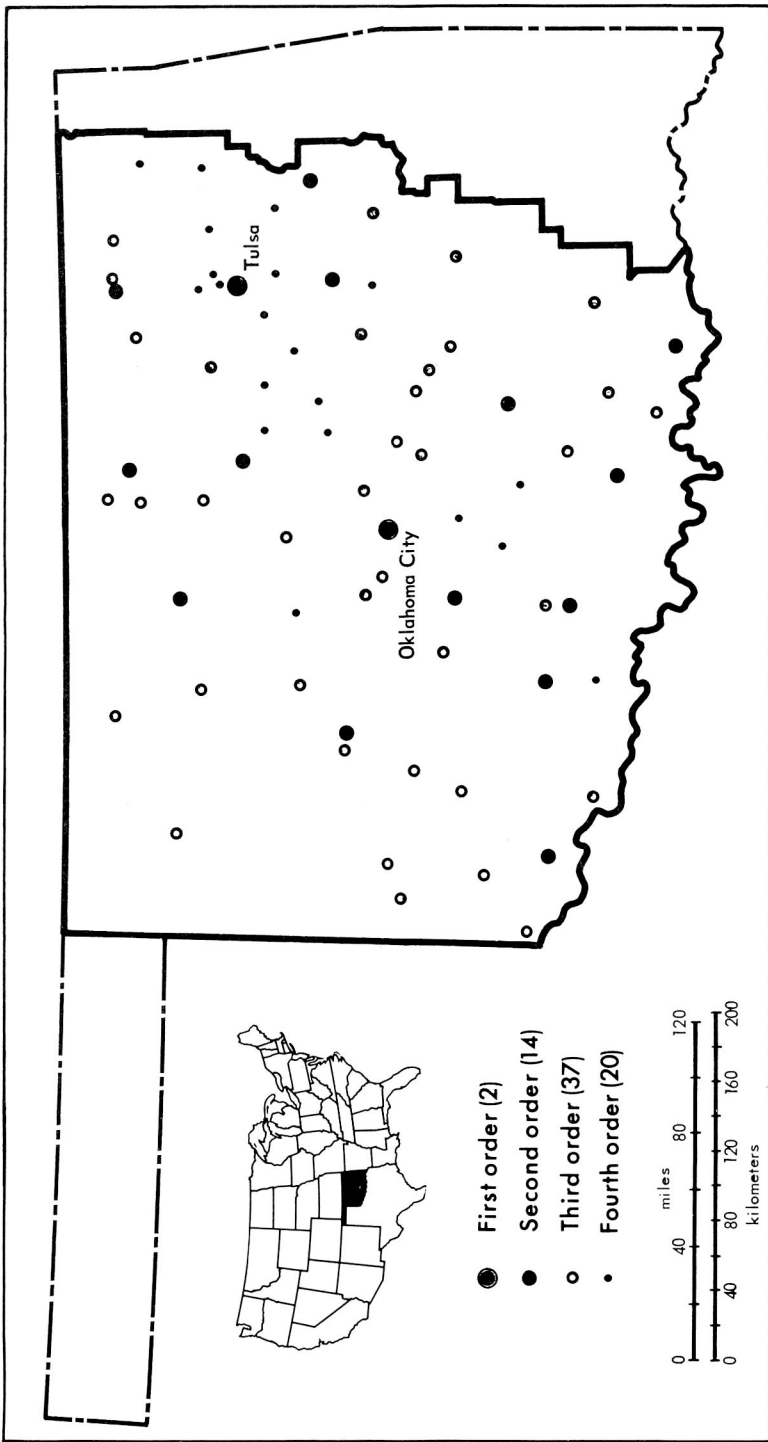
Figure 1. Hierarchy of Cities Within the Study Area

TABLE II. ROTATED COMPONENT STRUCTURE MATRIX

No.	VARIABLE Short Title	I	II	III	IV	V	VI	VII	VIII	IX	X	h²
2	Labor force	.984										.995
13	Retail establishments	.981										.996
1	Population	.980										.995
14	Retail sales	.980										.994
15	Food stores	.973										.989
16	Furniture stores	.971										.992
28	Towns within 25 miles	.652	—.493									.825
32	Rural population density		—.876									.890
34	Area in farms		.848									.812
33	Rural labor force in primary industry		.836									.818
29	Distance to city		.650									.738
35	Average farm size		.639								.569		.765
5	Under poverty level			—.858								.842
31	Rural income			.832								.792
6	City income			.716								.788
8	Aged 65 years and over			—.570			—.429	.484				.897
10	Persons per household			.550		.434		—.436				.943
30	Percent county population rural			—.520								.715
26	3-34 year olds in school				—.872							.798
25	Labor force in education				—.820							.863
9	Fertility ratio				.819							.804
7	Aged 17 years and under			.479	.649							.866
20	Public administration					.848					.441	.816
3	1960-70 growth					.736						.803
24	Finance and insurance						.832					.850

Table II (cont.)

No.	VARIABLE Short Title	I	II	III	IV	COMPONENT V	VI	VII	VIII	IX	X	h²
18	Airline stops	.423					.668					.771
21	County seat						.454			.435		.817
12	Professional services							.709				.573
17	Transportation							—.486				.627
4	Female labor force							.410				.722
11	Wholesale and retail								.835			.831
22	Manufacturing								—.691			.808
27	Construction								.421			.664
19	Distance to highway									.835		.828
23	Communications										.738	.669
	Variance	6.925	4.028	3.930	3.079	2.062	2.033	1.973	1.745	1.576	1.542	
	Percent total variance	19.78	11.51	11.23	8.80	5.89	5.81	5.64	4.98	4.50	4.41	
	Cumulative percent total variance	19.78	31.29	42.52	51.32	57.21	63.02	68.66	73.64	78.14	82.55	

loadings on population, labor force, and retail sales and establishments. This appears to be a general index of functional size, similar to that isolated by Berry (1972, p. 17). The second component serves to distinguish rural farm/non-farm differences. Percent of county area in farms and primary industry among the rural labor force have strong positive loadings, and rural population density loads negatively. Overall, this may be viewed as analogous to the surface characteristics used by King (1961) and others (O'Farrell, 1968, 1970). Towns with high scores on this component are those in the less urbanized western portion of the study area. The third factor indexes higher income towns in the more prosperous rural areas, and also correlates with several of the social-demographic variables included in the analysis. Conceptually, this appears to be related to one type of surface-center interaction in the study area, especially that of income variation. A single aspect of urban function is identified by the fourth component. School enrollment and employment in education load negatively, while the fertility ratio loads positively. Component scores serve especially to identify college towns within the system.

The fifth component is again related to function, correlating positively with employment in public administration, and also with intercensal growth. The sixth dimension loads primarily on employment in the finance, business and insurance sectors. Medium-sized cities score highly on this component. To some extent it is an index of higher-order services. The seventh component is related especially to professional service employment, another sector of the economy. The eighth dimension is also related to function, reflecting the inverse relationship ($r = -.47$) between employment in trade and employment in manufacturing industry. Scores on this component index the primary focus of each center in this respect. The two remaining components each account for less than six percent of the common variance: the ninth correlates with distance from the interstate highway, and the tenth with employment in communications.

Together the ten dimensions form a comprehensive and detailed statement of each center's size, economic specialties, and surface characteristics, and appear to embrace all features of the system which might be related to spacing.

The Urban Hierarchy

Before proceeding with the test of the relationship between size and spacing, it is necessary to derive groups of cities. Thomas' model postulated a relationship between the size of each center and the distance separating it from its nearest neighbor of population size equal to, or greater than, itself. Such a test is not possible in the present case since each city is characterized by scores on ten dimensions. Therefore, we derive groups of cities which, hopefully, are hierarchical in nature, then

use as the criterion the distance separating each city from its nearest neighbor of a similar or higher hierarchical level.

Using standard scores on each of the ten components, hierarchical groups were obtained by the use of Ward's (1963) clustering procedure.[5] Distance measures in ten-dimensional Euclidean space were used to join cities in a pairwise fashion in order to minimize the objective function of within-group sums-of-squares. Grouping was terminated when no single city remained unallocated; at this stage the within-group sum-of-squares accounted for 76 percent of the total. Multivariate tests showed the four groups obtained to be significantly different (Tatsuoka, 1971). Wilks' likelihood-ratio criterion was 0.0042.[6]

The nature of the groups was examined by discriminant analysis, since it was necessary that functionally hierarchical groups be obtained. Three significant functions were obtained: the first two correlated strongly with components one and six (correlations are respectively —.782 and —.693). These components were originally interpreted as functional size and higher order functions, so it is clear that the clusters were functionally differentiated. A plot of the four groups on the two components shows the distinct clusters, and also their hierarchical ordering (Figure 2). Thus the four clusters were interpreted as levels on the urban hierarchy. Two first-order centers were identified: Oklahoma City and Tulsa. There were 14 second-order cities, 37 third-order and 20 fourth-order centers. It is clear that the fourth level was incomplete: the lower limit of 2,500 population necessitated by data availability probably truncated this group and eliminated the possibility of lower levels being included.

City Spacing and the Urban Dimensions

The distribution of cities of the four levels is shown in Figure 1. Straight-line distance in miles was measured between each city and its nearest neighbor of similar or higher hierarchical level. To some extent, "typical" distances are associated with each level (Table 3). These distances reinforce the interpretation of the groups of cities as hierarchical, since functionally lower centers should be separated by shorter distances.

Stepwise regression was used to test the relationship between distance and the component scores. Since the components are all related to size

TABLE III. HIERARCHICAL LEVELS AND MEAN DISTANCES

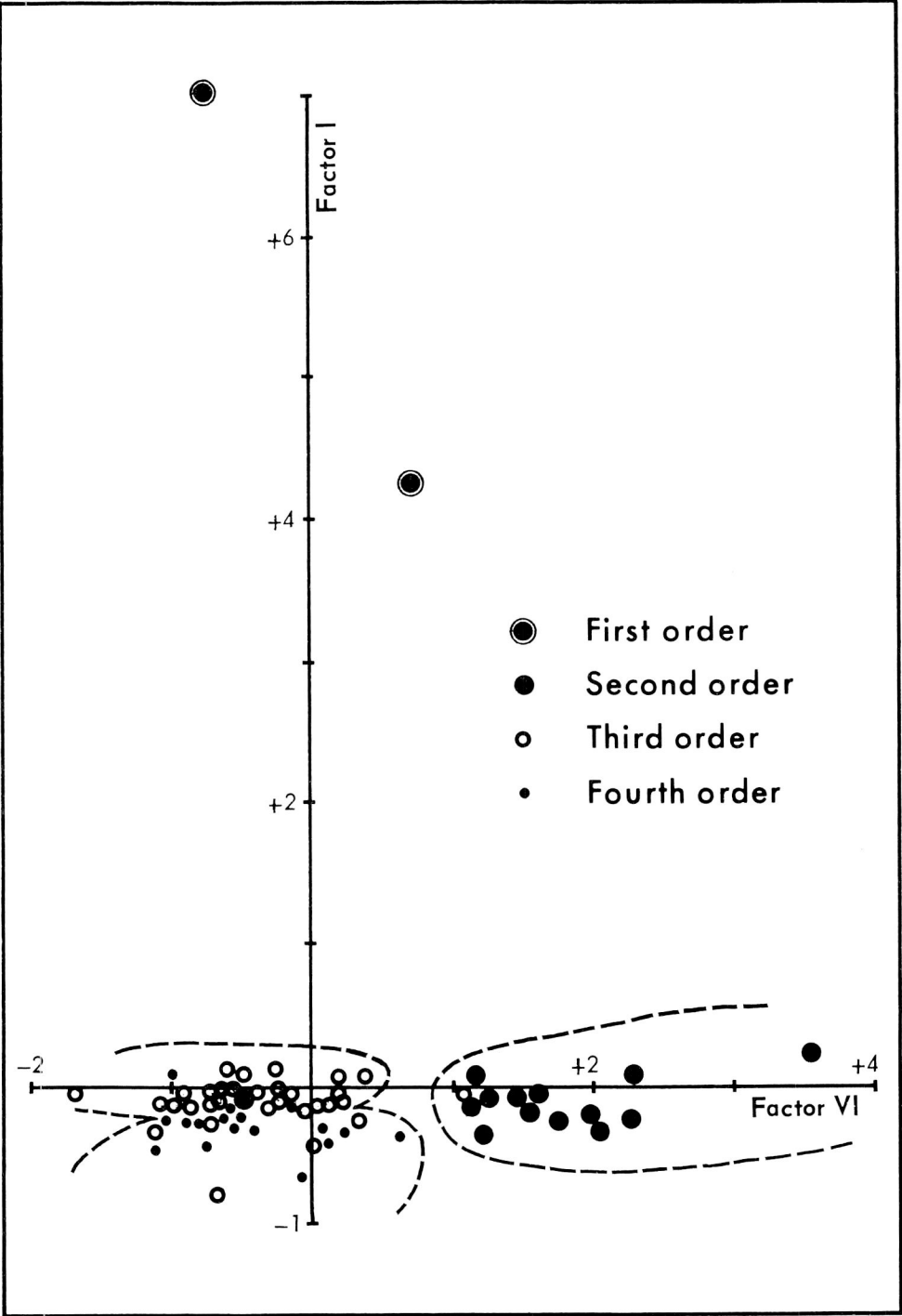| Level	N	Mean	Std. Dev.
1	2	109.0	——
2	14	42.29	11.79
3	37	19.14	9.57
4	20	14.90	5.59

Figure 2.   Clusters of Cities with Regard to Components I and VI

TABLE IV. STEPWISE REGRESSION ANALYSIS

Variable	Definition	Transformation
Dependent	Distance in miles of a city from its nearest neighbor of similar or higher hierarchical level.	None
Independent	Scores of a city on each of ten ortho-gonal factors derived from an R-mode principal components analysis.	Standardized

Multiple R	0.8884
Multiple R Squared	0.7892
Intercept Value	25.2599
Std. Error of Est.	9.7301

Order of entry	Variable Entered	Coefficient	Multiple R Squared	Increase in R Squared
1	1	14.0378	0.5077	0.5077
2	6	8.2281	0.6827	0.1750
3	4	—3.8136	0.7203	0.0376
4	9	3.3210	0.7488	0.0286
5	8	2.8943	0.7705	0.0217
6	3	2.1726	0.7827	0.0122*
7	2	1.4760	0.7883	0.0056*
8	5	—0.5445	0.7891	0.0008*
9	7	—0.1881	0.7892	0.0001*
10	10	—-0.1301	0.7892	0.0000*

*Not significant at the .05 level.

or surface characteristics, a strong relationship was hypothesized, with each dimension contributing additively to the level of explanation achieved.[7] The results are shown in Table 4. The first predictor to enter the equation is the central services factor, which accounts for half the variation in the distance variable. This level of explanation is comparable to the results of past studies using both population size and central services as predictors (Thomas, 1961; O'Farrell, 1968). The sixth component, which identified higher-order services and accounted for only 5.8 percent of the original variance in the data matrix, explains an additional 17.5 percent of the spacing variation. However, the only other independent variable to explain as much as three percent is the fourth component, which identified college towns. With all ten variables included, the explanatory power of the model is almost 79 percent.

The best predictor appears to be the functional size of each center. This is directly analogous to Christaller's importance-centrality concepts in the marketing system, so there is a strong theoretical basis for this relationship. The second variable — the finance and business factor — explained a significant amount of variation in distance, without a strong

theoretical basis, since, although it does include tertiary functions, it is primarily quaternary-level activity. Its association with distance may be a widespread phenomenon, or it may be limited to the present study area. Alternatively, other specific sets of functions may replace it in other areas. The third factor to enter the equation is interesting, although its explanatory powers are low. This is the educational function, which should theoretically have no relationship to distance, but its inclusion in the regression model for the Oklahoma system proves otherwise. The fourth variable, component nine, indexes the growth associated with accessibility to the interstate highway system. To some extent this may be interpreted as a K=4 type variable. The failure of the second and third components — the surface-type dimensions — to achieve some explanation is surprising. It was expected that these would operate to remove surface distortions which affected function-spacing relationships. This may be due to the relative smallness and homogeneity of the study area, or to the variables chosen to operationalize the concept.

Conclusions

Overall, this approach to the analysis of city spacing patterns appears promising. The use of components analysis to decompose the functional size and surface characteristics of an urban system into its basic dimensions provided deeper insight into the system than would be available otherwise. The use of cluster analysis to obtain a functional hierarchy was validated, and the regression model was quite powerful. The relationships revealed by the model included both conventional and innovative relationships. The predictive power of the central services component was expected, while that of the higher-order service and educational functions has not been shown before. Moreover, King (1966) has shown the stability of dimensions of this type over time, so longitudinal studies of component-spacing relationships should be feasible. This may afford insight into dynamic aspects of spacing.

On the other hand, the model also shows the need for finer calibration, involving the use of more detailed employment data, in order to identify economic sub-sectors which are related to distance (Parr, 1973). Given this, and replication in different areas, this approach may produce inductive generalizations about the spacing of cities which could contribute substantially to urban theory.

FOOTNOTES

[1]For nearest-neighbor analyses of city spacing, see, for example: King (1962) and Dacey (1962). Measures of system entropy are used in Medvedkov (1967), and Semple, *et al.* (1970).

[2]As a case in point, the hierarchy identified by Brush (1953) in southwestern Wisconsin has been analyzed by Dacey (1962) by nearest-neighbor methods and through entropy techniques by Medvedkov (1967). Conflicting results were obtained. For a discussion of hierarchies, see Davies (1966) and Marshall (1969).

[3]See, for example, the indices developed by Olsson, *et al.* (1963) and Davies (1965).

[4]For discussions of the social correlates of city size, see Schnore, *et al.* (1955) and Hawley (1971, pp. 135-138).

[5]A distinction is required between the *functionally hierarchic* groups obtained and the *hierarchic clustering* algorithm itself. Taxonomically, the algorithm is hierarchic since successive clusters are obtained by the merging of previous clusters. For a discussion, see Johnson (1967).

[6]Bartlett's chi-square approximation of lambda for overall significance was 355.749 with 30 degrees of freedom. This is significant at the .001 level.

[7]Since the components are orthogonal, the regression model is truly additive. Multicollinearity has been a problem in previous studies: see, King (1961) and O'Farrell (1970).

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